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Transition of low-temperature plasma similarity laws from low to high ionization degree regimes

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Abstract

Similarity laws are often employed when characteristics of two or more discharge plasma systems are compared. The classical similarity laws were previously validated and applied for weakly ionized plasma discharges. However, the classical similarity laws will not be valid for all plasma regimes. Especially for strongly ionized regimes, scaling laws are not well elaborated. In this study, transition characteristics of low-temperature plasma similarity laws are evaluated from low to high ionization degree regimes. The similarity relations of plasma density and ionization degree are obtained for geometrically similar gaps. The deviations from classical similarity laws are observed, which are gradually enlarged as the ionization increases from low to high degrees. The transition characteristics are affected by the significance of nonlinear reaction processes (such as three-body collisions), resulting in that the classical similarity relations hold up to a higher ionization degree at low pressure than that at high pressure. The time-dependent scaling characteristics of species densities and electron kinetic behaviors in geometrically similar gaps are also examined. The results are beneficial for utilizing the similarity laws in a wide range of ionization degree regimes, which is essential for correlating plasma characteristics in geometrical similar vessels of various scales.

Keywords: gas discharge, similarity law, ionization degree, low-temperature plasma, scaling law, electron density

1. Introduction

Similarity laws of gas discharge physics have been historically derived and continuously developed to correlate fundamental characteristics among two or more compared plasma systems [1–5]. Based on the similarity law theory, under certain conditions physical parameters in geometrically similar plasma vessels of different dimensions can be linearly scaled with dimensional factors [6–9]. Or equivalently, the plasma discharges can be characterized based on combined parameters for multiple similar discharge systems. For weakly ionized low-temperature plasma discharges, the similarity laws were previously found to be competent in geometrically similar gaps at low pressures, which could be used to extrapolate the discharge characteristics from a known one to others [10–12]. In recent years, low-temperature plasma applications have been expanding to ultra-large or small scales, which inevitably enter a completely new regime compared to the traditional cases. The similarity laws can be employed to either understand or predict the plasma characteristics based on the scaling relations which might be crucial in designing new plasma devices. The classical similarity laws require that the inherent physical processes in different gaps are the same, which might not always be valid especially when the discharge plasma enters strongly ionized regimes. Therefore, the validity and possible violations of the similarity relations should be carefully noted before their practical applications. Previously, the violations of the scaling laws were partially discussed in [13–18] and modified similarity relations were proposed in theory for different plasma density regimes [19–22]. However, the applicability and the critical regimes of the different similarity relations have not been straightforwardly characterized up to now. In which regime the classical similarity relations still hold and how the similarity relations should be modified in a target regime are still ambiguous, which require more systematic investigations.

The factors for violating similarity laws can be generally classified into two aspects. One aspect is from the plasma chemistry side, which includes linear and nonlinear processes from the perspective of the similarity laws [3, 4, 23–26]. In the discharge processes, elastic collisions, one-step ionizations, and Penning ionizations are typical linear processes. The examples of the nonlinear processes are three-body collisions, stepwise ionizations, and photo-ionizations. In some regimes, the impacts of the nonlinear processes might be less important, which ensures the validities of the similarity laws in the corresponding regimes. Note that we use the terms 'linear' and 'nonlinear' processes rather than the previously used 'allowed' and 'forbidden' processes for clarity [4, 17]. The second aspect comes from the transportation of the plasma species and energy, which are also classified as linear processes, such as drift and first-order diffusion, or nonlinear ones, such as heat transfer and nonlocal effect of energetic electrons [17]. The validity of similarity laws is generally determined by the competition of the significance of linear and nonlinear processes in specific regimes.

In this paper, the validity of similarity laws in different ionization degree regimes is studied, and the inherent causes for the similarity deviations are illustrated, correspondingly. Using a global (spatially averaged) plasma model incorporating fundamental reactions, the transition characteristics of similarity laws are evaluated from low to high ionization degree regimes. The similarity relations of the plasma species densities and ionization degrees in geometrically similar gaps are investigated at both low and high pressures. The time-dependent similarities of the species densities and the electron energy distributions in geometrically similar systems are presented. The principal motivation for this study is to seek essential strategy to estimate bulk plasma characteristics based on geometrical similarity of vessels in different scales. The results are beneficial for understanding and utilizing the similarity laws in a wide range of plasma ionization degree regimes.

2. Similarity laws and model description

2.1. Similarity laws

Similar discharges are usually identified when discharge currents are the same with equal potential differences in different gaps. The similarity theory defines the conditions under which similar discharges may occur in similar gaps [23, 27]. The similar gaps, having different dimensions and containing the same gas at different pressures, are usually geometrically similar and maintain the products of gap dimension and gas pressure same, i.e. $p_i d_i = p_j d_j$, where p_i and p_j are gas pressures and d_i and d_j are gap dimensions. According to the similarity principles, discharge characteristics in multiple gaps could be characterized by using combined parameters [4]. Typical examples include (i) Paschen's law describing the breakdown voltage V_b as a function of pd (gas pressure × gap dimension), i.e. $V_b = f(pd)$ [28–34], and (ii) Townsend's ionization coefficient α as a function of E/n_g (electric field over gas number density), i.e. $\alpha = f(E/n_g)$ [1, 35–38].

With the same external circuit and the same electron energy distributions at the corresponding points of space, similar discharges may occur among multiple gaps and the plasma density n_{pi} and n_{pj} in the ith and the jth gap can be scaled as follows

$$n_{pi} = (k_j/k_i)^m \cdot n_{pj} = k^m \cdot n_{pj}, \tag{1}$$

where k_i and k_j are the dimensional scaling factors between compared gaps and if the ith gap is chosen as the basis case, we have $k_i = 1$ by default and $k = k_j/k_i = d_j/d_i$; m is a factor depending on the discharge regime and $m = 2$ is one special case for weakly ionized plasmas. According to the gap dimension relation and the same mean velocity of the particles, the time interval in the ith and the jth gap scales as $k = \mathrm{d}t_j/\mathrm{d}t_i$. Therefore, in two completely similar discharges, the relationship of the charged species production rate $\mathrm{d}n/\mathrm{d}t$ in the gap volume is expressed as

$$(\mathrm{d}n/\mathrm{d}t)_i = k^{m+1}(\mathrm{d}n/\mathrm{d}t)_j, \tag{2}$$

which is usually employed to check the linearity of the reaction processes [4]. A specific process is linear if equation (2) is satisfied; otherwise, it is nonlinear. The detailed derivation of the plasma densities from classical similarity laws can be found in [3, 20, 23]. Note that the linearity of the processes could be different depending on the discharge regimes. For highly ionized plasma discharges, m should be modified due to the violation of the classical similarity laws. It can be generally extrapolated that if the gases in two similar gaps are both approaching the fully ionized state, we have

$$\lim_{X\to 1} [n_{pi}/n_{pj}] = p_i/p_j = d_j/d_i = k, \tag{3}$$

where X is the ionization degree (fractional density of ionized neutrals). Even though a full ionization is rarely reached for low-temperature plasmas, equation (3) indicates that when the two discharge systems approach the fully ionized state, the classical scaling of $m = 2$ will not be satisfied and the limit $m = 1$ will be reached as the ionization degree increases. We distinguished the density scaling laws with different m values and called them k^2 scaling, i.e. $n_{pi} = k^2 n_{pj}$, and k scaling, i.e. $n_{pi} = k n_{pj}$, with $m = 2$ and 1, respectively. Equation (3) is consistent with the scaling laws for high-density plasmas which was proposed by Muehe in [19].

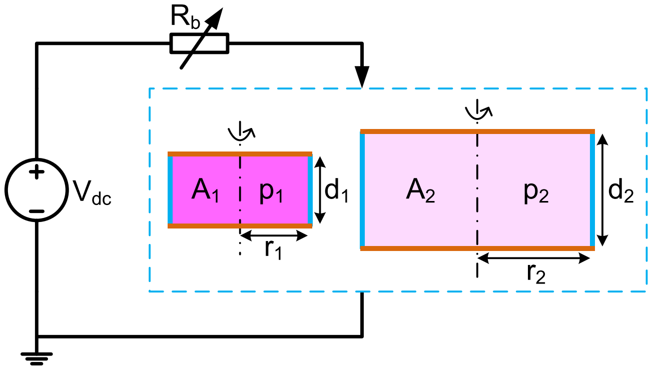

Figure 1. Schematic of the global model. Space-averaged plasmas in cylindrical gaps A_1 and A_2 are connected to an external circuit with direct-current voltage source V_{dc} and ballast resistor R_b. A_1 and A_2 are two geometrically similar ($d_1/r_1 = d_2/r_2$) cylindrical gaps. p_1 and p_2 are gas pressures in gaps A_1 and A_2 with $p_1d_1 = p_2d_2$ and $p_1r_1 = p_2r_2$ to satisfy similar discharge conditions.

The scaling relation for the ionization degree in the two gaps can be obtained by

$$X_i = n_{pi}/n_{gi} = k^m \cdot n_{pj}/n_j \cdot p_j/p_i = k^{m-1}X_j, \qquad (4)$$

where $n_{gi} \propto p_i$ and $n_{gj} \propto p_j$ are neutral gas number densities in the ith and the jth gap; $m = 2$ and $X_i = kX_j$ according to the classical similarity laws. If the ionization degree approaches unity, from equation (4) we will have

$$\lim_{X\to 1} [X_i/X_j] = k^{m-1} = 1, \qquad (5)$$

where the limitation $m = 1$ is reached. Note that when the ionization degree approaches the fully ionized state, the plasma will be in a Coulomb-collision dominated regime. However, the Coulomb collision is a binary elastic collision between two charged particles interacting through their own electric fields. Coulomb collisions will not exaggerate the violations of similarity relations since all the elastic collisions are linear in terms of the classical similarity laws. Note that equation (5) is also consistent with the so-called B-similarity since it can be obtained by keeping the Boltzmann equations invariant [19, 22].

It can be extrapolated that as the plasma ionization degree increases, the similarity relations may also change, resulting in a transition of plasma similarity laws from low to high ionization degree regimes. This phenomenon can be confirmed based on the relations of plasma parameters and by checking the significance of fundamental processes in geometrically similar gaps.

2.2. Model description

As shown in figure 1, the volumetric plasmas in two geometricallcally similar gaps (A_1 and A_2) coupled with the same external circuit are employed to investigate the scaling relation of the similarity laws. The geometry of the two gaps is cylindrical with $d_1/r_1 = d_2/r_2$, where d_1 and d_2 are the gap distances; r_1 and r_2 are the cylinder radius. p_1 and p_2 are the gas pressures in gaps A_1 and A_2 with $p_1d_1 = p_2d_2$ and $p_1r_1 = p_2r_2$ to fulfill similar discharge conditions. V_{dc} is the applied voltage and R_b is the ballast resistor. The top and bottom surfaces of the cylinder are treated as two parallel electrodes (anode and cathode), carrying the discharge current I_p. The gap voltage V_p is obtained consistently through $V_p = V_{dc} - I_pR_b$. By adjusting the resistor R_b, the discharge's operating point and the ionization degree can be adjusted to different regimes.

The simulations are conducted with argon at 300 K. Five species are included: electrons (e), atomic ions (Ar^+), molecular ions (Ar_2^+), excited atoms (Ar^*), and the background ground state atoms (Ar). The plasma reactions include (R1) momentum: $e + Ar \to e + Ar$, (R2) excitation: $e + Ar \to e + Ar^*$, (R3) de-excitation: $e + Ar^* \to e + Ar$, (R4) one-step ionization: $e + Ar \to 2e + Ar^+$, (R5) stepwise ionization: $e + Ar^* \to 2e + Ar^+$, (R6) recombination: $Ar^+ + 2e \to Ar + e$, (R7) recombination: $Ar_2^+ + e \to Ar^* + Ar$, (R8) three-body collision: $2Ar + Ar^+ \to Ar + Ar_2^+$, (R9) quenching: $Ar^* + 2Ar \to 3Ar$, (R10) associative ionization: $Ar^* + Ar^* \to Ar_2^+ + e$, and (R11) charge transfer: $Ar_2^+ + Ar \to Ar^+ + 2Ar$. Among all the reactions, R1, R2, R4, and R11 are linear processes according to the classical similarity law and others are nonlinear [26]. It can be extrapolated that if the classical similarity law holds, the effect of nonlinear processes could be eliminated. If the similarity law first holds and then becomes invalid, the deviation can be attributed to the significance of the included nonlinear reactions since other nonlinear mechanisms, such as gas heating and thermal ionization, are not considered. The quasi-neutral plasma condition is enforced, and the electron density can be obtained by summing the ion densities. The rate coefficients for electron-impact reactions are obtained from the Boltzmann equation solver (BOLSIG+ in the two-term approximation) [39–41]. The species continuity equations are time-dependently solved with local field approximations for mean electron energy. The diffusion loss of the charged particles to the dielectric wall is approximated by an equivalent volumetric loss and the characteristic diffusion length Λ is quantified as $\Lambda = \sqrt{(\pi/d)^2 + (2.405/r)^2}$ for a cylindrical gap with a radius r and a length d [29]. It is worth noting that the diffusional loss is a linear process and will not contribute to the deviations of the classical similarity laws. In the simulations, the nonlinear processes are not isolated and their significance are self-consistently evaluated across different discharge regimes, which could be tuned by external resistors through controlling system currents [42, 43]. In this work, when the gap dimensions and the gas pressure are chosen for geometrically similar gaps, V_{dc} is fixed at 1000 V and the working point of the discharge is adjusted by changing the resistor R_b to ensure the plasma sustaining from low to high ionization degree regimes.

3. Results and discussion

According to the *pd* scaling law, the characteristic lengths of the low-temperature plasma could range from centimeters to meters at low pressure while reduce to micrometer to

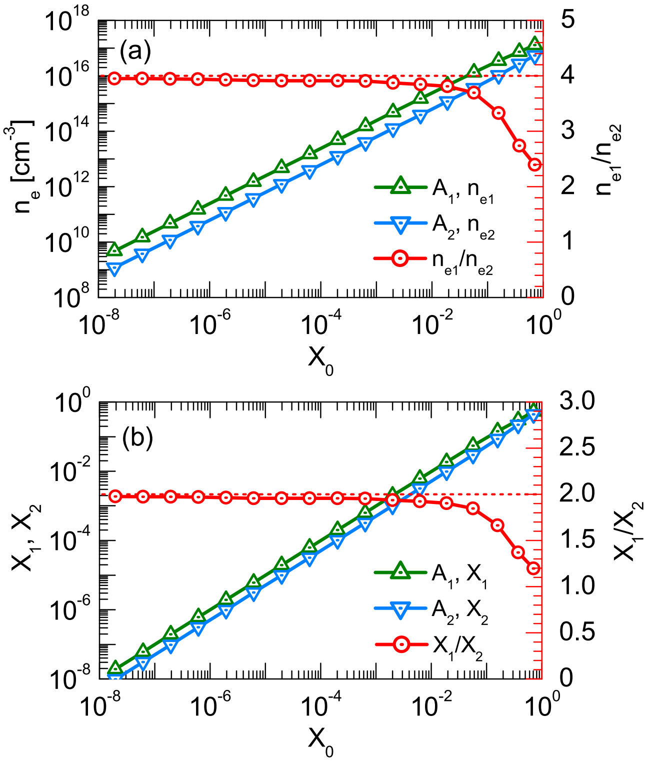

Figure 2. Low-pressure cases: A_1: $p_1 = 7.6$ Torr, $d_1 = 10$ cm, $r_1 = 10$ cm and A_2: $p_2 = 3.8$ Torr, $d_2 = 20$ cm, $r_2 = 20$ cm. (a) Electron densities and their ratio and (b) ionization degrees and their ratio as a function of X_0.

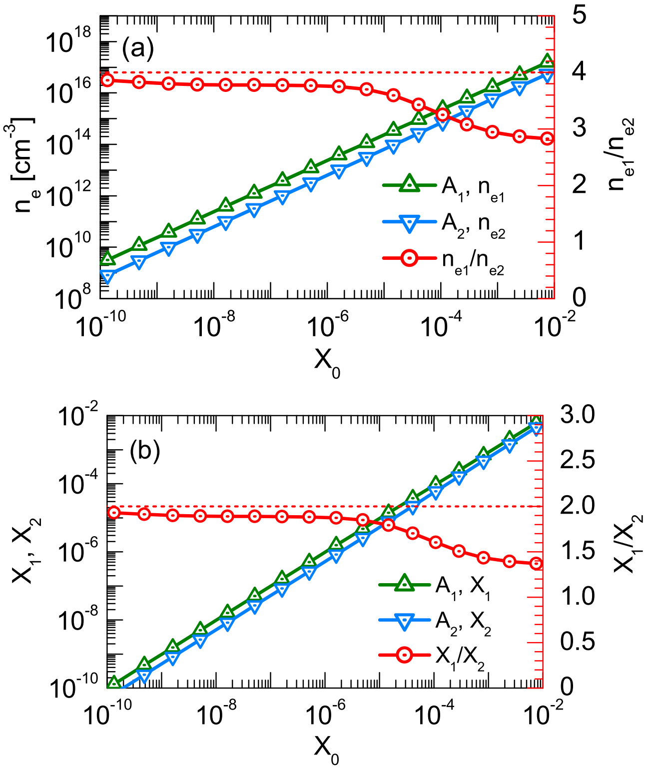

Figure 3. High-pressure cases: A_1: $p_1 = 760$ Torr, $d_1 = 0.1$ cm, $r_1 = 0.1$ cm and A_2: $p_2 = 380$ Torr, $d_2 = 0.2$ cm, $r_2 = 0.2$ cm. (a) Electron densities and their ratio and (b) ionization degrees and their ratio as a function of X_0.

millimeter scales at high pressure to avoid arc discharges [23, 26]. The simulations were carried out in two gaps at low pressures at first since the similarity laws are mostly applied in this pressure regime. The parameters in the two geometrically similar gaps are: $p_1 = 7.6$ Torr, $d_1 = 10$ cm, and $r_1 = 10$ cm for gap A_1; $p_2 = 3.8$ Torr, $d_2 = 20$ cm, and $r_2 = 20$ cm for gap A_2. The electron density and the corresponding ionization degree, as well as their corresponding ratios, at the steady state are obtained and plotted as a function of ionization parameter X_0, as shown in figure 2. The parameter X_0 is an averaged scaled ionization degree which is defined as

$$X_0 = \frac{1}{N}\sum_{i=1}^{N} k_i^{m-1} \cdot X_i, \tag{6}$$

where k_i and X_i are the scaling factor and the ionization degree in the ith gap, respectively; N is the total number of the compared gaps and here $N = 2$. The defined parameter X_0 provides a weighted average ionization degree with all the gaps considered. Since m changes for different ionization degree regimes, for simplicity, $m = 2$ can be used in equation (6) to observe deviations from the classical similarity laws. In figure 2(a), with the gap dimension and gas pressure fixed, the plasma density increases as the ionization degree X_0 increases. According to the classical similarity laws, equation (1), the scaling factor between A_1 and A_2 is $k = d_2/d_1 = 2$ and the electron density ratio should be $k^2 = 4$. It is shown that with a low ionization degree the classical similarity relation (k^2 scaling) generally holds up to $X_0 \approx 10^{-2}$, beyond which the deviation becomes obvious. In figure 2(b), as X_0 increases, the ionization degree ratio shows a similar deviation tendency and approaches unity, which is consistent with the k scaling prediction. It is worth noting that for low pressures, the classical similarity laws usually hold in a wide range of ionization degree since the critical value of X_0 where the deviation occurs is relatively high. The included nonlinear processes (such as three-body collisions) are not significant, which will not violate the classical scaling relations in the low ionization degree regime, while in the high ionization regimes the parameter ratios are determined by the background gas number density, resulting in the k scaling. The transition characteristics are confirmed as the plasma operates from low to high ionization degree regimes.

Figure 3 shows the scaling relations of the plasma density and the ionization degree at high pressures when the nonlinear processes, such as the three-body collisions, might be significant. The parameters in the two geometrically similar gaps are: $p_1 = 760$ Torr, $d_1 = 0.1$ cm, and $r_1 = 0.1$ cm for gap A_1; $p_2 = 380$ Torr, $d_2 = 0.2$ cm, and $r_2 = 0.2$ cm for gap A_2. In figure 3(a), it is observed that a minor deviation from theoretical prediction, which is more obvious compared to the low-pressure case, exists across the ionization degree range from 10^{-10} to 10^{-6} and the violation of the classical similarity laws becomes more severe when the ionization degree is above $\sim 10^{-5}$. In figure 3(b), a similar deviation tendency of the ionization degree ratio is observed as X_0 increases. Compared

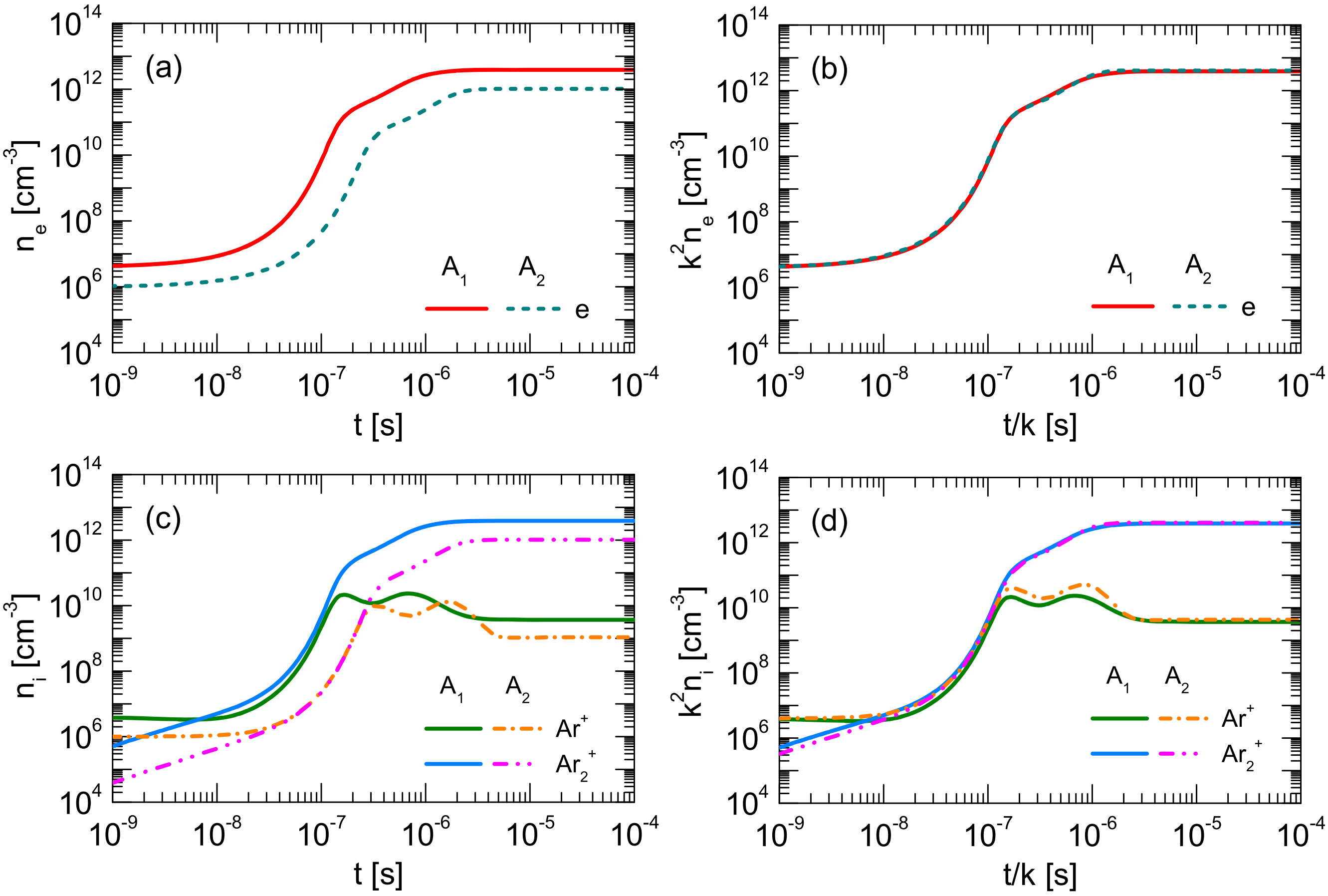

Figure 4. The time-dependent evolution and scaling of the densities of charged species in gaps A_1 ($p_1 = 760$ Torr, $d_1 = 0.1$ cm, $r_1 = 0.1$ cm) and A_2 ($p_2 = 380$ Torr, $d_2 = 0.2$ cm, $r_2 = 0.2$ cm). (a) Electron density versus time; (b) Scaled electron density versus scaled time; (c) Ion densities versus time and (d) Scaled ion densities versus scaled time. In this case, $X_0 = 1.70 \times 10^{-7}$.

to the results in figure 2, the similarity laws will be better satisfied at low pressures than at high pressures even though the scaling factors are the same. The significant violation of the similarity law occurs with a lower ionization degree at the high pressures than at the low pressures. The different transition characteristics might be affected by the importance of nonlinear reaction processes, such as three-body collisions, which are sensitive to the gas pressure regimes. As mentioned above, the linearity of a reaction process from the similarity laws could change from one ionization degree regime to another, which can be judged from equation (2). As the ionization degree increases from low to high, m changes from 2 to 1 and the requirement of the production rates scaling varies from k^3 to k^2 according to equation (2). In this situation, the electron and metastable densities between two geometrically similar gaps both transform with the scaling factor k according to equations (1) and (2) [22, 26]. The relationships of the charged species production rate from the stepwise ionization as well as the ion-electron volume recombination will automatically satisfy equation (2) with $m = 1$. Thus, the stepwise ionizations and the ion-electron volume recombination act as nonlinear processes according to the k^2 scaling in weakly ionized regimes while becoming linear processes in term of the k scaling in strongly ionized regimes. However, the three-body collisions are nonlinear in both regimes, which could be very sensitive to gas pressure regimes. The differences of the transition characteristics in figures 2 and 3 are mostly due to varying contributions from the nonlinear processes with dependence on gas pressure.

In order to check the validity of time-dependent similarity laws with a lower X_0 at high pressures when the steady-state similarities are confirmed, the temporal evolutions and the density scaling of the plasma species in geometrically similar gaps with the scaling factor $k = 2$ are shown in figure 4. In figure 4(a), the electron density in both gaps A_1 and A_2 reaches a steady state in a similar way but with different temporal transitions. In figure 4(b), the scaled electron density $k^2 n_e$ and the scaled time t/k are presented, with k being a scaling factor of each gap. It is observed that the scaled temporal evolutions of the electron densities are overlapping, which indicates that the discharge behavior in gaps A_1 and A_2 could be extrapolated from one to the other based on the classical similarity laws in this regime. Figure 4(c) shows the evolutions of the atomic and molecular ion densities and figure 4(d) shows the corresponding scaled evolutions of the ion densities. It can be seen that the atomic ions are dominating ions at the early stage and their density becomes orders smaller compared to the molecular ions at the steady-state, which is due to the strong impact of the three-body collisions which consume the atomic ions and convert

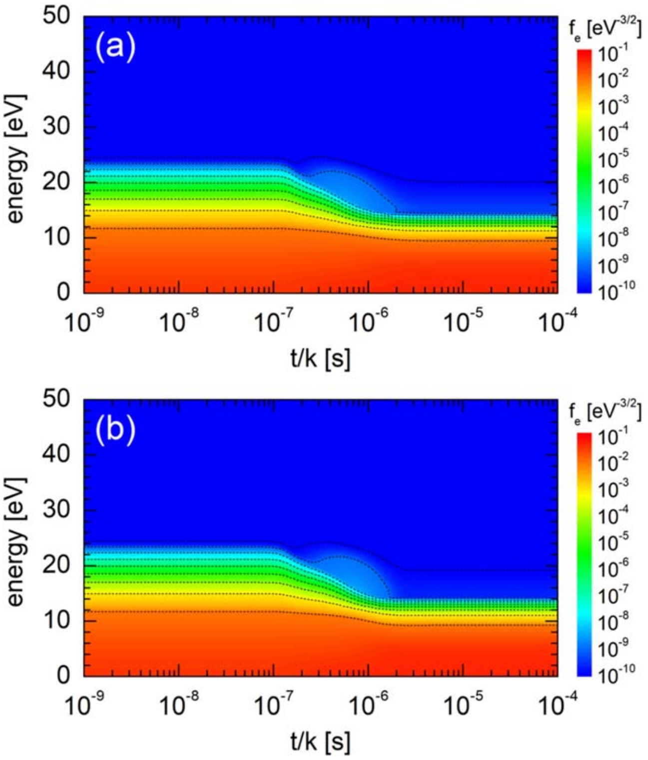

Figure 5. The time-dependent evolution and scaling of the electron energy distribution functions. (a) Time-dependent EEPF in A_1 ($p_1 = 760$ Torr, $d_1 = 0.1$ cm, $r_1 = 0.1$ cm) and (b) time-dependent EEPF in A_2 ($p_2 = 380$ Torr, $d_2 = 0.2$ cm, $r_2 = 0.2$ cm). In this case, the scaling factor is $k = 1$ for A_1 and $k = 2$ for A_2.

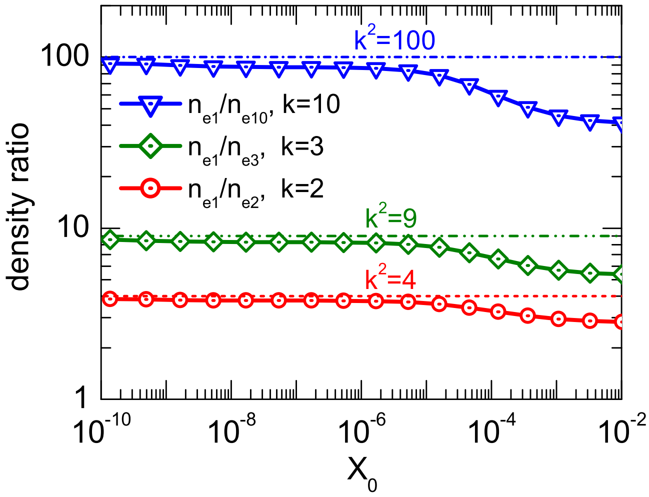

Figure 6. The plasma density ratio versus X_0 with different scaling factors. n_{e1} is chosen as the base case, and the corresponding scaling factors are $k = 2$ for n_{e1}/n_{e2}, $k = 3$ for n_{e1}/n_{e3}, and $k = 10$ for n_{e1}/n_{e10}.

them to the molecular ions. As shown in figure 4(d), the scaled densities for the atomic and molecular ions are almost the same, which confirms the similarity of time-dependent scaling of the individual ion densities. Therefore, even though the nonlinear processes are included, the classical similarity relations for the charged species density are still valid at high pressures with a small scaling factor ($k = 2$) in the low ionization regime.

According to the previous definition of the similarity laws, the electron energy distributions at the corresponding space position in similar gaps should be the same, which was also confirmed by Margenau in [27]. More strictly, it was proposed that the invariant distributions functions are required for all particles, which indicates the distribution function of each species in similar discharges should be correspondingly the same [19, 22]. The time-dependent electron energy distribution functions in the two geometrically similar gaps are also compared, as shown in figure 5. The time-dependent electron energy distributions shown correspond to the case with $X_0 = 1.70 \times 10^{-7}$ in figure 4. The electron energy distributions are initialized to be stationary and then solved time-dependently by the Boltzmann solver. The obtained electron energy probability functions (EEPFs) f_e in $eV^{-3/2}$ are normalized with electron density and scaled with the factor k in time. The electron energy distribution shows an obvious transition between 10^{-7} and 10^{-6} s, resulting in a significant truncation of the high-energy tail at the steady state, which is due to the formation of the plasma and decrease of the electric field. The appearance of the bump-like tail in the EEPFs is due to the impact of the superplastic reactions as the excitation degree (fractional density of excited neutrals) increases [41]. In general, the temporal evolutions of the electron energy distributions observed are quite similar, which in turn verifies the proposition of invariant distributions of individual species for achieving similar discharges.

Figure 6 shows the transitions of the similarity laws with different scaling factors. Four geometrically similar gaps are assumed with $p_i d_i = p_j d_j$ and $p_i r_i = p_j r_j$, where i and $j \in \{1, 2, 3, 4\}$, $i \neq j$. The gap A_1 with $p_1 = 760$ Torr, $d_1 = 0.1$ cm, and $r_1 = 0.1$ cm is chosen as the base case. n_{e1} is the electron density in gap A_1 and chosen as the baseline. The corresponding scaling factors for the density ratio are $k = 2$ for n_{e1}/n_{e2}, $k = 3$ for n_{e1}/n_{e3}, and $k = 10$ for n_{e1}/n_{e10}. In all cases, the deviations from classical scaling laws are observed as the ionization degree increases from low to high values. The absolute deviations of the density ratio increase as the scaling factor increases even in the low ionization regime. The tendency of the similarity relation for electron density transiting from k^2 to k scaling is confirmed with dependence on the ionization degree of the plasma. For low degrees of ionization, the k^2 scaling is usually applied to the discharge, such as glow discharge, which was extensively confirmed by experimental studies [4, 15, 44–46]. In the case of strongly ionized states, the k scaling will apply but only for the quasi-neutral region in the discharge, excluding the cathode fall layer with space charge near the electrode, or for cases when the role of cathode fall layer is less important [22]. A typical example for the application of the k scaling is the positive column in a glow discharge, which was confirmed by the external characteristic (total current scaling) by experiment previously [19, 22, 47, 48]. Since the global model is used, the results are not limited to glow discharge regimes but generally apply for low-temperature plasmas. Even though the practical situations might be more complicated if the spatially dependent plasmas are examined, the present results confirm the qualitative trends of the transition of the similarity laws from low to high ionization degree regimes. It can be generally understood that for the k^2 scaling, the deviations might be affected by the nonlinear processes or the scaling factor while approaching the fully ionized state, the k scaling comes into play since the plasma density is ultimately determined by the neutral gas number density. Therefore, if the ionization degree keeps increasing, the classical similarity laws will be eventually violated and the high ionization state similarity laws will be reached, resulting in the transition phenomena, as shown in figures 2, 3 and 6, regardless of gas pressure regime or scaling factor.

4. Conclusions

The transition characteristics of the low-temperature plasmas similarity laws from low to high ionization degree regimes are studied in geometrically similar gaps. The deviations of the classical similarity laws are confirmed as the ionization degree increases at both low and high pressures. At low pressures, the critical ionization degree for the classical similarity laws being valid is higher than that at high pressures. For the high-pressure cases, in the low ionization degree regime the classical similarity laws can still be valid with a small scaling factor even though the nonlinear processes are included. With the classical similarity laws being valid in the low ionization regime, the time-dependent scaling of the species densities and the electron energy distributions are also examined. The tendencies of the similarity relation for electron density transiting from k^2 to k scaling as the ionization degree increases are consistent with theoretical predictions. The transition characteristics of the similarity laws are affected by the varying contribution of nonlinear reaction processes. However, with the consideration of spatially dependent transport mechanisms, such as plasma convection and local structures (cathode layers with either positive or negative space charge effect), the transition phenomena would become more complicated, which requires further numerical and experimental investigations. Future research also includes quantifying the divergence from the classical similarity law prediction in the high ionization degree regime, and tracing the contribution of each type of nonlinear reactions during the transition.

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