Enhancement of high-order harmonic generation in intense laser interactions with solid density plasma by multiple reflections and harmonic amplification

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High order harmonic generation (HHG) from laser solid-density plasma interactions is considered as a promising way to generate bright and ultrashort burst of x-rays. One of the key issues for HHG is the low conversion efficiency due to a power law fall off in harmonic intensity. Here, we explore a possible mechanism for HHG enhancement by reflecting an intense laser pulse multiple times between two plasma surfaces. We find that HHG can be nonlinearly enhanced by orders of magnitude through multiple reflections compared with a single reflection. We argue this enhancement is due to the non-linear combination of the relativistically oscillating plasma mirror and the harmonic-rich pulse generated in preceding reflections. This mechanism should allow moderate power laser systems to efficiently generate coherent, ultrashort bursts of soft x-rays. © *2015 AIP Publishing LLC*. [http://dx.doi.org/10.1063/1.4916739]

High order harmonic generation (HHG) from laser solid-density plasma interactions has been studied extensively[1–14] due to its capability of generating phase-locked bursts of harmonics into x-rays. The relativistic oscillating mirror mechanism[1,3,4] occurs when the surface electrons of the target are oscillated collectively by an intense laser field to relativistic speeds, such that a periodic Doppler shift due to the moving point of reflection gives rise to HHG of the laser frequency.

Previous theoretical studies in HHG typically focused on the electron dynamics on interfaces[1–4] and the scaling of the harmonics strength with drive laser intensity. The effect of the plasma density gradient was also recently studied.[10,12] However, the efficiency of HHG from a single laser surface interaction is still very low, especially with ultraclean surface conditions[12] or more moderate intensities. Thus, it is of interest to seek alternative configurations for efficient HHG.

In this letter, we investigate the effect of multiple reflections of the laser pulse on HHG and show that the intensity of the high order harmonics can be nonlinearly enhanced, that is, increased beyond that from linearly adding the harmonics from each reflection. A schematic is shown in Fig. 1. This could be achieved in practice by propagating an intense laser pulse into a narrow capillary[15,16] at an angle of incidence $\theta = \pi/4$, for example. The rational is that by reflecting a laser pulse multiple times from two adjacent plasma surfaces, the harmonic components generated at one reflection may be enhanced at the subsequent reflections, providing a higher overall efficiency for HHG. By using particle-in-cell (PIC) simulations, we demonstrate that HHG can be enhanced by orders of magnitude from multiple reflections. We argue that this enhancement is due to the non-linear dynamics of plasma mirror motion, which is driven by the harmonic-rich pulse, as a result of the preceding reflections. This is supported by comparing HHG from single reflection of a pristine pulse (of frequency ω_0) with that of a pulse with one more harmonic component (of frequencies ω_0 and $n\omega_0$, where $n > 1$ is an integer), based on both PIC simulations and simple analytical model. This mechanism, which enhances HHG by reflecting an intense pumping laser pulse superimposed on a weaker harmonic pulse from a single solid plasma surface, may be considered as another effective method for HHG enhancement.

The simulations are performed by using a one-dimensional (1D) PIC code, EPOCH,[17] where all the three components of the fields and velocities are used. The simulation region is of 20 μm in length, where the two plasma walls, both 4 μm thick, are separated by a distance of 12 μm. A 10 fs (FWHM intensity) 800 nm Gaussian pulse, preloaded in the center of the simulation box, gets reflected back and forth between the two

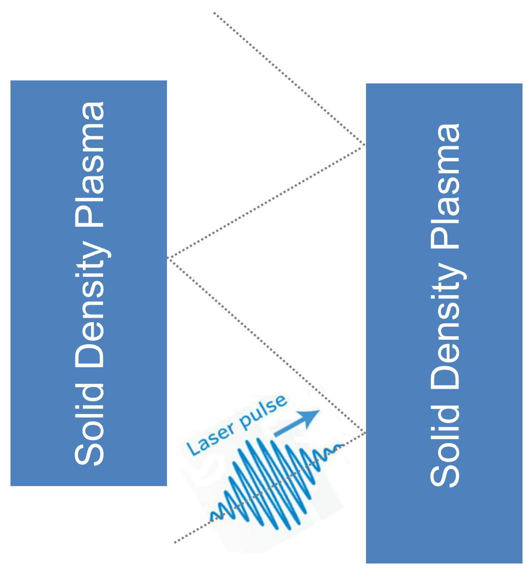

FIG. 1. A laser pulse is reflected back and forth between two overdense plasma surfaces to enhance HHG.

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plasma surfaces. The laser is at normal incidence and is linearly polarized, with peak intensity within $10^{19} < I < 10^{21}$ W cm^{-2}, corresponding to a normalized vector potential within $2.2 < a_0 < 22$. A cell size of 4 nm is used, with 320 particles per cell. Open boundaries are used, from which any reflections would be eliminated. Ions are fixed, which means that the surface remains steep for every bounce (as if it was a new surface each time, as in reality). The plasma walls have a density in the range of $50n_c - 500n_c$, where $n_c = 1.746 \times 10^{21}\,\mathrm{cm}^{-3}$ is the classical critical plasma density for the 800 nm laser. Similar behaviors on HHG enhancement are observed for various combinations of laser intensity and plasma density.

The reflected pulse contains components of high harmonics of the fundamental laser frequency, created through the relativistic oscillating mirror effect[1–4] whenever the pulse hits the surfaces. Fig. 2 shows an example of HHG by multiple reflections. The initial laser pulse is shown in Fig. 2(a). The peak intensity is 5.3×10^{19} W cm^{-2} (corresponding to an electric field $E_y = 2 \times 10^{13}$ V/m). Both plasma walls have a density of $150n_c$. Figs. 2(b)–2(e) show the evolution of the reflected laser pulse after different numbers of reflections m from the two plasma surfaces. The arrows in the figures indicate the direction of pulse propagation. It is clear that as m increases, the reflected pulse becomes more and more distorted from its initial sinusoidal shape, implying richer harmonic contents are generated in the pulse. Note the plasma surface oscillation during one reflection greatly damps out before the laser pulse comes next time, therefore has little effect on the subsequent reflections.

Figure 2(f) shows the Fast Fourier Transformed (FFT) intensity spectrum of the laser pulse in Figs. 2(a)–2(e). The initial laser pulse ($m = 0$) has only one peak at the laser frequency $\omega = \omega_0$. After one reflection ($m = 1$), at least four harmonic peaks are presented, but with relatively low amplitudes. Note that only odd harmonics (e.g., $\omega/\omega_0 = 3, 5, 7, \ldots$) are present in the spectrum, in agreement with the selection rules[1] for linearly polarized laser with normal incidence. In this case, the radiation source term is the transverse electron current along the mirror surface, $J_y = \rho p_y/\gamma$. To the first order, the transverse momentum $p_y \propto A_y$, which is the laser field of frequency ω_0, the charge density ρ, and the relativistic factor γ are determined by the ponderomotive force, which oscillates with frequency $2\omega_0$. Therefore, J_y contains only odd harmonics, so does the reflected radiation.[18] The harmonics are significantly enhanced by the subsequent reflections. After four reflections ($m = 4$), the 7th and 9th harmonics are enhanced *by almost 3 orders of magnitude*, as compared to the single reflected pulse ($m = 1$). Furthermore, higher order harmonics that are absent in the early stages, such as $\omega/\omega_0 = 17$, 19, 21, and even 25, start appearing in the spectrum after the 4th reflection. Note that in Fig. 2(f), there is a sudden drop in the amplitude peaks beyond $13\omega_0$. This is due to the fact that the solid plasma ($150n_c$) becomes transmissive to radiation with frequency of $\sqrt{150}\omega_0 \cong 12.25\omega_0$ or higher. The reflected pulse therefore has reduced harmonics peaks beyond $13\omega_0$, since these harmonic components escape the cavity at each reflection. The complicated structure of the HHG spectra around the cut-off harmonic is probably due to the highly nonlinear electron dynamics near the plasma surface.

Figure 3(a) shows the amplitude of the FFT intensity spectrum of the laser pulse in Fig. 2, as a function of time and frequency ω/ω_0. The ticks of the horizontal axes mark the beginning of each reflection, when the laser pulse reaches one of the plasma surfaces. The number of harmonics increases after each reflection, meanwhile, the amplitude of the existing harmonics also increases. After ten reflections, the maximum visible harmonic appears in the spectrum reaches $\omega/\omega_0 = 29$. The amplitude of the harmonic peaks is plotted as a function of m in Fig. 3(b). The harmonic peaks are enhanced by orders of magnitude by subsequent reflections, and eventually saturate for these conditions after about ten reflections.

We argue that this HHG enhancement is due to the nonlinear dynamics of plasma mirror motion, which is driven by the superposition of pulses with frequencies at harmonics of the fundamental frequency ω_0, produced from the preceding reflections. This is supported by comparing HHG from single reflection of a pristine pulse (of frequency ω_0) with that of

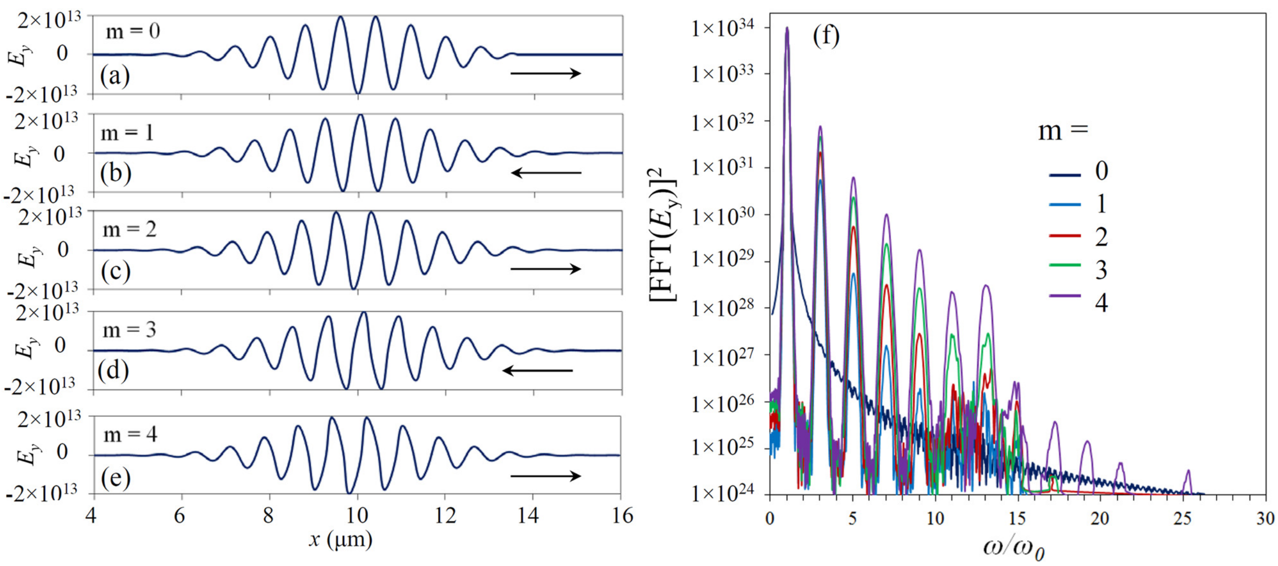

FIG. 2. (a)–(e) The electric field E_y (in V/m) of the laser pulse after different number of reflections $m = 0 - 4$. (f) Intensity spectrum, $[\mathrm{FFT}(E_y)]^2$ of the laser pulses in (a)–(e).

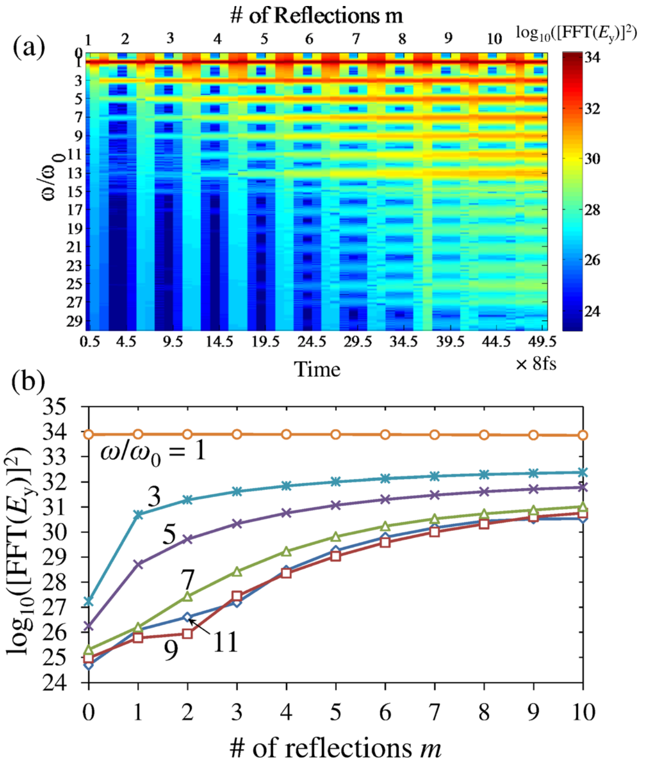

FIG. 3. (a) The intensity spectrum of the laser pulse in Fig. 2, as a function of time (as well as number of reflections m) and frequency ω/ω_0. The ticks of the horizontal axes mark the beginning of each reflection. The lighter colored columns following the ticks approximately correspond to the duration of pulse reflection from one of the plasma surfaces. (b) The amplitude of the harmonic peaks as a function of m.

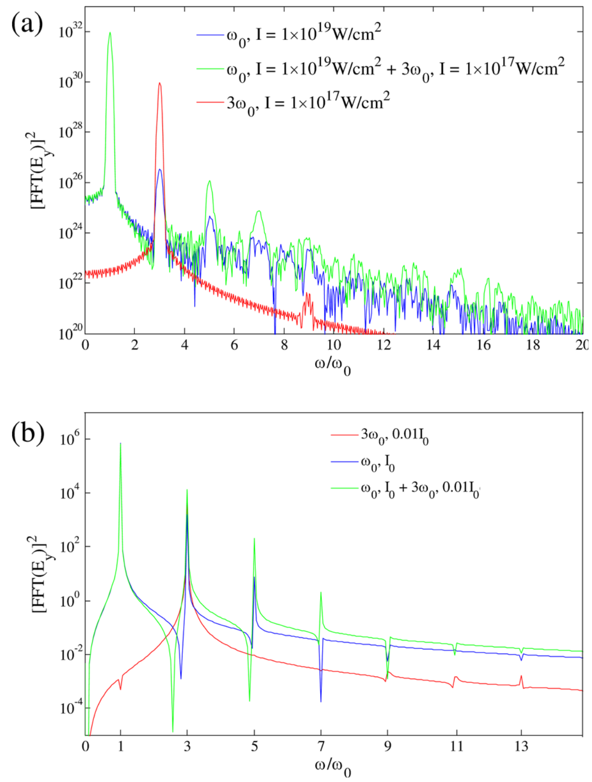

FIG. 4. Intensity spectrum of laser pulses after single reflection for three different incident pulses, from (a) 1D PIC simulation, with incident pulse of ω_0 and $I = 1 \times 10^{19}\,\mathrm{W\,cm^{-2}}$, $3\omega_0$ and $I = 1 \times 10^{17}\,\mathrm{W\,cm^{-2}}$, and superposition of the two, (b) analytical calculation, Eqs. (1)–(3), with incident pulse of ω_0 and I_0 (normalized to 1), $3\omega_0$ and $0.01I_0$, and the superposition of the two. $A_m = 0.1$ is used in Eq. (1).

the superposition of the pristine pulse with one more harmonic component (of frequencies ω_0 and $n\omega_0$), from both PIC simulations and simple analytical model.

Figure 4(a) shows, from 1D PIC simulations, the intensity spectrum of the pulse after reflection from single plasma surface, for three different cases of incident pulses: (1) A Gaussian pulse with frequency ω_0 ($\lambda = 800\,\mathrm{nm}$) of peak intensity $I = 1 \times 10^{19}\,\mathrm{W\,cm^{-2}}$, (2) a Gaussian pulse with $3\omega_0$ and $I = 1 \times 10^{17}\,\mathrm{W\,cm^{-2}}$, and (3) a combined pulse consisting of a superposition of the pulses in (1) and (2). In case (3), the 5th ($\omega/\omega_0 = 5$) and 7th ($\omega/\omega_0 = 7$) harmonics are enhanced by at least one order of magnitude as compared to case (1), which were not even present in case (2). The 9th harmonic in case (3) is also enhanced as compared to that in both case (1) and case (2). This indicates that a dramatic increase in harmonic intensity occurs only for the combination of pulses, and cannot be attributed to the pulses individually.

HHG via laser-solid interaction may be modeled by a simple oscillating mirror model for normal incidence,[2,3] which assumes that the electrons at the plasma surface undergo forced oscillations around the edge of an immobile step-like ion background driven by the ponderomotive force of the incident laser. Its origin is the $\vec{v} \times \vec{B}$ term of the Lorentz force, and it varies as $F_p(t) \sim I_0\lambda_0^2 \sin(2\omega_0 t)$. The electrostatic field due to charge separation serves as the restoring force. Thus, to first order, the trajectory of the mirror motion can be assumed as, $X_m(t') = A_m \sin(2\omega_0 t' + \phi_m)$. The harmonic components in the reflected pulses are due to the retardation effect between the point of reference (the observer) and the electron sheet interface from which the incident wave is reflected. Assuming that the laser field initially at the observer position is $E_{in}(t) = \sin(\omega_0 t)$. When it returns to the observer position (after reflecting from the moving mirror surface), the wave form will become $E_{re}(t) = \sin\{\omega_0[t + 2X_m(t')/c]\} = \sin(\omega_0 t + 2k_0 X_m(t'))$, where $t' = t + X_m(t')/c$ is the retarded time, c is the speed of light, and k_0 is the laser wave number in free space.

Consider an incident laser pulse with two frequency components, $E_{in}(t) = \sin(\omega_0 t) + s\sin(n\omega_0 t)$, where s is the field strength at frequency $n\omega_0$ with respect to that at ω_0, and n is an integer. To the lowest order, the resulting mirror motion in its own frame would be

$$X_m(t') = A_m \sin(2\omega_0 t') + A_m \frac{s^2}{n^2} \sin(2n\omega_0 t'), \tag{1}$$

$$t' = t + X_m(t')/c. \tag{2}$$

Assuming $s \ll 1$, the second term of Eq. (1) will be negligible and the surface will primarily oscillate at ω_0 only. Due to the nonlinear dynamics of the mirror motion, the reflected wave becomes

$$E_{re}(t) = \sin(\omega_0 t + 2k_0 X_m(t')) + s\sin(n\omega_0 t + 2nk_0 X_m(t')), \tag{3}$$

which contains enhanced harmonic components. By solving Eqs. (1)–(3) self consistently, we obtain the FFT intensity spectrum of the reflected pulse, as shown in Fig. 4(b), for $n = 3$ and $s = 0.1$ (intensity ratio of 0.01). The main features (such as the relative strengths and locations of the harmonic peaks) of the HHG spectrum from the PIC simulation (Fig. 4(a)) are reasonably predicted by the analytical model, showing evidence of harmonic amplification from a multifrequency incident pulse.

We further propose that this harmonic amplification mechanism, which enhances the higher harmonics by reflecting an intense pumping laser pulse superimposed with a weaker harmonic pulse from a single solid plasma surface, may be considered as another effective way for HHG enhancement.

In this letter, we studied alternative ways to enhance HHG at intense laser solid-density plasma interaction. We found that reflecting a laser pulse multiple times back and forth between two plasma surfaces can enhance the HHG by orders of magnitude. This enhancement is due to the nonlinear dynamics of the plasma mirror motion, driven by the multifrequency pulse from proceeding reflections. This is verified by comparing the HHG from single reflection of an incident pulse of the fundamental frequency only with that of containing both fundamental frequency and one harmonic, via both PIC simulations and analytical theory. The multifrequency incident pulse gives much enhanced HHG. The latter enhances the higher order harmonics by reflecting an intense pumping laser pulse superimposed with a weaker harmonic pulse from a single plasma surface and may also be regarded as another effective way for HHG enhancement. Note that although the normal incidence considered here is not identical to Fig. 1, the 1D results relate the 2D if we consider an appropriate Lorentz boost in the direction of propagation.[19] The interaction at oblique incidence is left as an interesting subject for future work. The effects of issues, such as beam focusing, polarization, and incident angle, remain to be studied, perhaps using 2D or 3D models, and eventually in experiments.

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