Experimental validation of a higher dimensional theory of electrical contact resistance

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The increased resistance of a cylindrical conducting channel due to constrictions of various radii and axial lengths was measured experimentally. The experimental data corroborate the higher dimensional contact resistance theory that was recently developed. © *2009 American Institute of Physics*. [DOI: 10.1063/1.3205116]

Contact resistance is a topic of significant current interest to several fields: wire array z-pinches,[1–3] field emitters,[4,5] and high power microwave devices.[4,6] In these systems, poor electrical contact prevents efficient power coupling to the load, produces unwanted plasma, and even damages the electrodes. Contact resistance is also extremely important in wafer evaluation,[7] thin film resistors,[8] and metal-oxide-vacuum junctions.[9] Because of the surface roughness on a microscopic scale, true contact between two pieces of metal occurs only at the asperities (small protrusions) of the two contacting surfaces. Current flows only through these asperities, which occupy a small fraction of the area of the nominal contacting surfaces. This gives rise to contact resistance.[10–16]

While contact resistance is highly random, depending on the surface roughness, on the applied pressure, on the hardness of the materials, and perhaps most importantly, on the residing oxides and contaminants at the contact,[7,10,11] the basic model for contact resistance remains that of Holm.[10] Holm's model consists of two semi-infinite cylinders of radius b placed together. Current can flow through them only via a "bridge" in the form of a circular disk of radius $a \ll b$. This disk has zero thickness and is known as the "a-spot" in the literature. In the limit $b \to \infty$, Holm derived the expression for the contact resistance of the a-spot,[10]

$$R_c = \frac{\rho}{2a}, \quad \text{(Holm)}, \tag{1}$$

where ρ is the electrical resistance of the current channel. For finite values of b $(>a)$, Timsit[11] and Rosenfeld and Timsit[12] solved the Laplace equation for the a-spot geometry, and then synthesized their numerical results into a useful and accurate formula

$$R_c = \frac{\rho}{2a}\overline{R_{c0}}, \quad \text{(Rosenfeld and Timsit)}, \tag{2}$$

$$\overline{R_{c0}} = 1 - 1.41581(a/b) + 0.06322(a/b)^2 + 0.15261(a/b)^3 + 0.19998(a/b)^4. \tag{3}$$

Equation (2) reduces to Holm's result, Eq. (1), in the limit $b \to \infty$. When $a=b$, the contact resistance $R_c=0$ according to Eq. (3), a result expected of a uniform current channel.

Implicit in the theory of Holm[10] and Timsit[11] are two assumptions: (A) the a-spot has a zero axial length in the direction of current flow, and (B) the current channel is made of the same material, i.e., the effects of oxide have been ignored. Recently, Lau and Tang[16] generalized the Holm–Timsit theory by relaxing assumption (A), while retaining assumption (B).

Specifically, Lau and Tang[16] considered the a-spot to have a total axial length $2h$ [Fig. 1(a)], and proposed the following simple formula for the electrical contact resistance [cf. Eq. (15) of Ref. 16],

$$R_c = \frac{\rho}{2a}[\overline{R_{c0}} + s_c(h/a)], \tag{4}$$

$$s_c = (4/\pi)[1 - (a/b)^2]. \tag{5}$$

Equation (4) was derived using an intuitive physical argument and has been favorably compared with a numerical code. It has not been proven rigorously, however, as the analysis for this deceptively simple geometry [Fig. 1(a)] is exceedingly complicated[16] even for the ideal case where the current channel has a constant, uniform electrical conductiv-

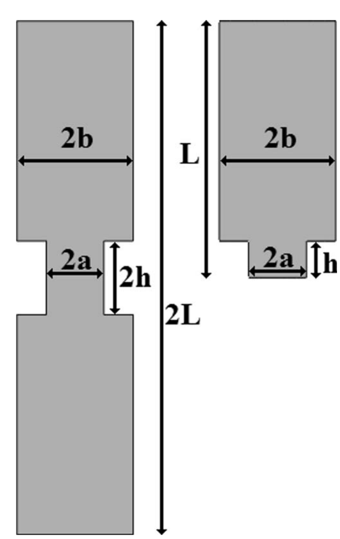

FIG. 1. (a) Left. A cylindrical current channel with main radius b, a constriction of radius a and axial length $2h$, and total axial length $2L$ $(\geqslant 2b, 2h)$. The Holm–Timsit model of a-spot is recovered in the limit $h=0$. (b) Right. The experimental setup, which is exactly one half of the theoretical model shown to the left.

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TABLE I. Dimensions, measured resistance, and expected resistance of the copper sulfate channels. All channels had diameter $2b=15.9\pm0.1$ mm, and length $L=40.8\pm0.2$ mm.

Resistor number	$2a$ (mm)(±0.1)	h (mm)(±0.2)	Measured R (Ω) (±0.1)	Experimental R_c (Ω) (±0.3)	Theoretical R_c (Ω)	Experiment/theory
1	N/A	N/A	182.0	0	0	N/A
2	8.1	1.5	225.0	86.0	74.1	1.16
3	8.0	3.2	250.4	136.7	125.2	1.09
4	8.0	7.3	299.7	235.3	228.7	1.03
5	8.0	11.4	344.0	323.9	330.8	0.98
6	8.0	15.4	389.0	414.1	440.1	0.94
7	4.2	1.4	361.2	358.5	312.9	1.15
8	4.1	3.0	451.3	538.6	514.2	1.05
9	4.1	5.0	636.0	908.1	765.1	1.19
10	4.2	7.1	697.0	1030	1002	1.03
11	4.1	9.1	824.8	1286	1279	1.01

ity. Equation (4) shows that the effect of finite h is to increase the contact resistance linearly with h, by an amount that is expected from the increase in the current path length associated with finite h, and from the decrease in the cross-sectional area in the channel constriction. Equation (4) reduces to Eq. (3) in the limit $h=0$.

Since Eq. (4) represents a first generalization of the Holm–Timsit theory to higher dimensions,[16] here we report the experimental validation of this theory. A solid piece of lucite was machined using various diameter mill and drill bits to obtain the geometry shown in Fig. 1(b), which is, by symmetry, half of Fig. 1(a). Ten channels of this general geometry (but with varying a's, and h's) were machined (dimensions listed in Table I). Channel dimensions were measured with a digital micrometer with 0.1 mm precision. An additional, baseline channel with no constriction was also machined (first row in Table I). All eleven channels were filled with a 36 g/l aqueous copper sulfate solution. Copper endcaps with conductive tabs were attached to both ends of the channel.

The resistance of each channel was measured to determine the increase in resistance as a function of constriction radius and height. The baseline channel without constriction was used to determine the resistivity of the solution. The resistivity of the copper sulfate solution used was 0.886 Ω m, which is many orders of magnitude higher than the resistivity of copper ($\sim10^{-8}$ Ω m). Thus we were able to assume that the current flow at the end caps was parallel to the axis of the cylinder, and this formed the basis of our symmetry arguments we used to compare Fig. 1(b) with Fig. 1(a). It was necessary to use the geometry in Fig. 1(b) to avoid trapping air bubbles at the constriction.

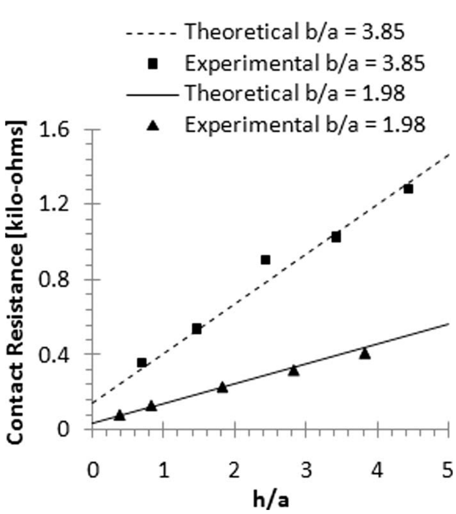

FIG. 2. The experimentally measured contact resistance and the theoretically predicted values. Five constriction heights (h) were used for each constriction radius (a).

The measured resistance of each channel is summarized in Table I. These values were doubled to account for the symmetry assumption and then twice the resistance of the nonconstricted channel was subtracted in order to determine the increase in resistance due to the constriction. Figure 2 compares these values with the theoretical curves described by Eqs. (4) and (5). The average ratio of the measured values to the predicted values for these ten cases is 1.06 ± 0.08. Thus, the experimental results match the proposed scaling law to within one standard deviation.

Among the various geometries for which the higher dimensional theory was developed,[16] the above experiments validated the scaling law for the case where the connecting bridge of the main current channel is cylindrical in shape. The data shown in Fig. 2 may also be considered as an indirect experimental verification of the Holm–Timsit theory of the a-spot, as when these data are extrapolated to $h=0$, the contact resistance indeed agrees with the classical theory of Holm and Timsit, Eq. (2). The experimental corroboration then provides some confidence to use the proposed scaling laws to calculate the contact resistance in response to pressure (thus linking the contact resistance to the hardness of the materials) and the Ohmic heating at such contacts. Despite this progress, the important effects of oxides on contact resistance remain to be quantitatively assessed.

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