

# A VOLTAGE SCALE FOR ELECTRO-THERMAL RUNAWAY\*

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## Abstract

A voltage scale,  $V_s$  that characterizes electro-thermal runaway, is deduced from the heat conduction equation,  $V_s = \sqrt{\kappa/\sigma'_0}$ , where  $\kappa$  is the thermal conductivity and  $\sigma'_0$  is the rate of change of the electrical conductivity with respect to temperature.  $V_s$  depends only on material properties and is independent of geometry and the operating voltage.  $V_s$  measures the intrinsic tolerance of the material to electro-thermal instability. Numerical values of  $V_s$  are consistent with the well-known properties of several common materials.

## I. INTRODUCTION

It has been estimated that contact problems account for approximately 40 percent of all electrical/electronic failures [1]. Severe heating due to local current constrictions at thin film contacts [2-4] and at bulk contacts [3,5] is also a concern in high power microwave sources, pulsed power systems, field emitters, thin film devices, integrated circuits, and interconnects. In this paper, we investigate one aspect of electro-thermal instabilities, namely, the increase in electrical conductivity as the temperature increases, as is typical of semiconductors. This may lead to thermal runaway, at a fixed voltage, as illustrated in Fig. 1.

## II. VOLTAGE SCALE FOR ELECTRO-THERMAL RUNAWAY

Consider the heat conduction equation,

$$C \frac{\partial T}{\partial t} = \kappa \nabla^2 T + \vec{J} \cdot \vec{E}, \quad (1)$$

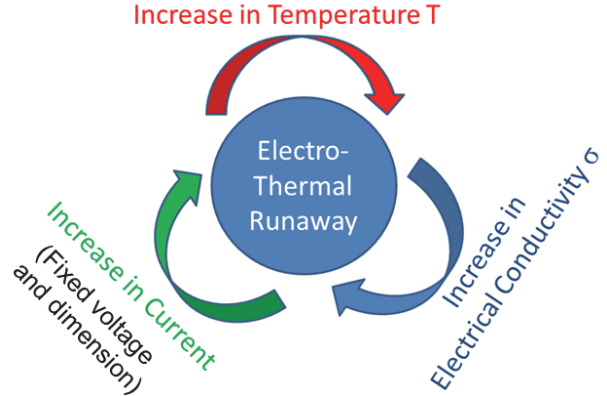


Figure 1. Electro-thermal runaway: a positive feedback.

where  $C$  is the heat capacity,  $\kappa$  is the thermal conductivity,  $T$  is the temperature relative to some base value,  $\vec{J}$  is the current density, and  $\vec{E}$  is the electric field. In Eq. (1), the first term on the RHS is the heat diffusion term, and the second term is the heat source term due to ohmic heating. Assume that electrical conductivity  $\sigma(T)$  increases as temperature  $T$  increases. By Taylor expansion and ignoring 2<sup>nd</sup> and higher orders, we have

$$\sigma(T) \approx \sigma_0 + \sigma'_0 T, \quad (2)$$

where  $\sigma_0$  is the initial conductivity of the material, and  $\sigma'_0$  is the rate of change of the electrical conductivity with respect to temperature. By using Eq. (2), the heat source term in Eq. (1) becomes,

$$\vec{J} \cdot \vec{E} = \sigma(T) E^2 = (\sigma_0 + \sigma'_0 T) E^2, \quad (3)$$

Inserting Eq. (3) into Eq. (1), we have,

$$C \frac{\partial T}{\partial t} = \kappa \nabla^2 T + (\sigma_0 + \sigma'_0 T) E^2. \quad (4)$$

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Taking the Fourier transform of Equation (4),  $T \sim e^{jk \cdot \bar{x}}$ , we have, with  $k = |\bar{k}|$ ,

$$\frac{\partial T}{\partial t} - sT = 2\pi \frac{\sigma_0}{C} E^2 \delta(\bar{k}), \quad (5)$$

where  $\delta(\bar{k})$  is the delta function and

$$s = \frac{\sigma_0 E^2 - \kappa k^2}{C}, \quad (6)$$

It follows that T grows exponentially with time if

$$E > kV_s \quad (7)$$

where  $V_s$  is the voltage scale which depends only on material properties,

$$V_s = \sqrt{\frac{\kappa}{\sigma_0}}. \quad (8)$$

Note that  $V_s$  is independent of geometry, the operating voltage, and the heat capacity. It measures the *intrinsic* tolerance of the material to electro-thermal instability.

Values of  $\kappa$ ,  $\sigma_0'$  and  $V_s$  are summarized in Table 1 for some common materials. We conclude that SiC is the most resistant to thermal runaway for the same geometry and the same operating voltage, consistent with the well-known property of this material.

**Table 1.** Voltage scale for electro-thermal runaway.

	$\kappa$ [W/(m-K)] [6-9]	$\sigma_0'$ [1/( $\Omega$ -m-K)] [10-13]	$V_s$ [Volt]
Si	142	0.0012-0.7	14.2-348.9
Ge	58	0.0001-0.05	34-761.6
C (graphite)	127	$1.67 \times 10^{-4}$ - $8.33 \times 10^{-6}$	872.9- 3903.8
SiC	370	$4 \times 10^{-7}$ or negative	$3 \times 10^4$

### III. SUMMARY

A voltage scale  $V_s$  is deduced from the heat conduction equation to characterize electro-thermal runaway. It depends only on material properties and is independent of geometry or the operating voltage. It measures the intrinsic tolerance of the material to electro-thermal instability. Our results based on the calculated voltage

scale  $V_s$  are consistent with the well-known properties of several common materials.

It is important to note that our formulation is quite general and may be applied to other materials, as long as proper approximations in Eq. (2) for the temperature dependent electrical conductivity are used.

### IV. REFERENCES

- [1] P. G. Slade, ed., *Electrical Contacts: Principles and Applications*, New York: Marcel Dekker, 1999; P. Stratton, private communications; *Review of Federal Programs for Wire System Safety*, National Science and Technology Council Final Report, 2000; J. S. Kuzniar and G. A. Slenski, "Wire Integrity Field Survey of USAF Legacy Aircraft", Defense Technical Information Center, ADP014075, 2001.
- [2] P. Zhang, Y. Y. Lau, and R. S. Timsit, "On the Spreading Resistance of Thin-Film Contacts", *IEEE Trans. Electron Devices*, vol. 59, pp. 1936 – 1940, 2012.
- [3] P. Zhang, "Effects of Surface Roughness on Electrical Contact, RF Heating and Field Enhancement", doctoral dissertation, University of Michigan, Ann Arbor (2012).
- [4] P. Zhang, D. Hung, and Y. Y. Lau, "Current flow in a 3-terminal thin film contact with dissimilar materials and general geometric aspect ratios", *J. Phys. D: Appl. Phys.*, vol. 46, p. 065502, 2013; Corrigendum, *ibid*, vol. 46, p. 209501 (2013).
- [5] P. Zhang and Y. Y. Lau, "Constriction resistance and current crowding in vertical thin film contact", *IEEE J. Electron Dev. Soc.*, in the press, 2013.
- [6] [mtixtl.com/xtlflyers/SiC.doc](http://mtixtl.com/xtlflyers/SiC.doc)
- [7] <http://www.ioffe.ru/SVA/NSM/Semicond/Si/thermal.html>
- [8] <http://www.ioffe.ru/SVA/NSM/Semicond/Ge/thermal.html>
- [9] <http://www.jyi.org/issue/thermal-conductivity-measurements-of-graphite-samples/>
- [10] [http://library.thinkquest.org/10784/tempcoeffs\\_resistivity.html](http://library.thinkquest.org/10784/tempcoeffs_resistivity.html)
- [11] <http://hyperphysics.phy-astr.gsu.edu/hbase/Tables/rstiv.html#c1>
- [12] [http://www.microwaves101.com/encyclopedia/silicon\\_carbide.cfm](http://www.microwaves101.com/encyclopedia/silicon_carbide.cfm)
- [13] [http://cdn.intechopen.com/pdfs/21151/InTech-Recent\\_developments\\_on\\_silicon\\_carbide\\_thin\\_films\\_for\\_piezoresistive\\_sensors\\_applications.pdf](http://cdn.intechopen.com/pdfs/21151/InTech-Recent_developments_on_silicon_carbide_thin_films_for_piezoresistive_sensors_applications.pdf)