Short-pulse space-charge-limited electron flows in a drift space

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In this paper, the space-charge-limited (SCL) electron flows in a drift space is studied by including the effect of finite electron pulse length, which is smaller than the gap transit time. Analytical formulas are derived to calculate the maximum SCL current density that can be transported across a drift space under the short-pulse injection condition. For a given voltage or injection energy, the maximum current density that can be transported is enhanced by a large factor (as compared to the long-pulse or steady-state case), and the enhancement is inversely proportional to the electron pulse length. In drift space, the effect of pulse expansion is important at very short-pulse length, and the short-pulse enhancement factor is smaller as compared to a diode. The enhancement factor will be suppressed when the injection energy is larger than the electron rest mass, and effect of pulse expansion is less critical at relativistic energy. The analytical formulas have been verified by performing a particle-in-cell simulation in the electrostatic mode. © *2008 American Institute of Physics*. [DOI: 10.1063/1.2941490]

I. INTRODUCTION

Short-pulse electron bunches has been drawing continuous interests in many areas, such as free electron lasers (FELs),[1] laser wakefield electron acceleration,[2–4] ultrafast electron diffraction,[5] and ultrashort coherent x-ray radiation.[6] Ultrashort electron bunches less than 100 fs have been recently obtained by using a lower-power femtosecond laser to trigger electron emission from sharp field emitters.[7–10] In modern FELs and high power microwave devices, the electrons pulses extracted from the cathode will first experience an acceleration phase (i.e., velocity modulation) followed by a drifting phase (i.e., density modulation). For high power applications, the space-charge effects of the electron beam will become important in both the acceleration and drifting phases.

In the acceleration phase (like a diode), the classical Child–Langmuir (CL) law, which predicts the steady-state space-charge-limited (SCL) current density or the maximum current density that can be transmitted across a gap is well studied.[11–16] The one-dimensional (1D) classical CL law is

$$J_{\mathrm{CL}} = \frac{4\epsilon_0}{9}\sqrt{\frac{2e}{m}}\frac{V_g^{3/2}}{D^2}, \tag{1}$$

where e and m are the charge and mass of the electron, respectively, and ϵ_0 is the free space permittivity. While Eq. (1) is easy to derive, it is only recently that the 1D classical CL law is extended to a two-dimensional (2D) model with simulation results,[17] analytical solutions,[18] effects of nonuniform emission,[19] and a three-dimensional (3D) uniform model.[20] The classical CL law has also been extended to quantum regime to include the quantum effects on the dynamic of SCL electron flows in a nanogap.[21–25] The short-pulse CL law has been also studied in classical,[26,27] quantum, and weakly relativistic[28] regimes. Recent developments of the CL law in multidimensional models and quantum models can be found in two review papers, respectively.[29,30] Recently, the classical CL law has been extended to study the SCL electron flows in a dusty plasma diode filled with stationary charged dusty impurities.[31]

Similar to the 1D classical CL law, as shown in Eq. (1), the equivalent classical SCL current density in a drift space is[32]

$$J_d = \frac{32\epsilon_0}{9}\sqrt{\frac{2e}{m}}\frac{V_g^{3/2}}{D^2}, \tag{2}$$

which is eight times higher than the classical CL value. In comparison to the diode case, the dynamics of the SCL electron flow in a drift space is less studied. Some recent studies include the radiation power loss of the SCL electron beam in a closed drift tube,[33] and the transition of the 1D unrelativistic SCL electron flow to a magnetically limited flow in a cylindrical drift tube.[34] All the studies of the SCL electron beam in a drift space has been limited to the steady-state condition, where the effect of short electron pulse length (smaller than the gap transit time) is not studied.

In this paper, we present a 1D short-pulse SCL current model in a drift space for both classical (in Sec. II) and relativistic (in Sec. III) regimes, which serves as an extension of the short-pulse CL law.[26] When the electron pulse length is much shorter than the gap transit time, we will show that the enhancement factor formulated before for the diode[26] can not be directly applied to the drift space, where the enhancement factor for the latter case is predicted to be smaller in this paper. The influence of the relativistic effects at high injection energy (larger than the electron rest mass) will also be investigated. Note the calculations presented in this paper

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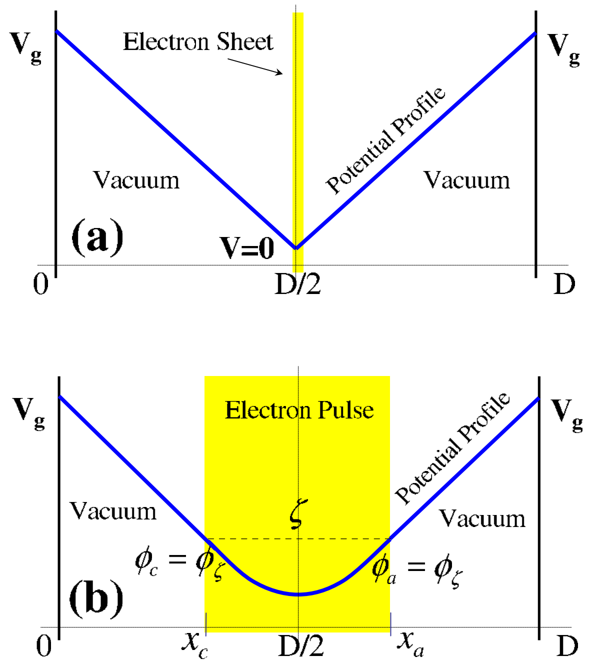

FIG. 1. (Color online) The diagram of a 1D drift space of gap spacing D in (a) a single electron sheet model, and (b) an equivalent drift space model with a short electron pulse having a width of ζ and a potential of ϕ_ζ at the beam-vacuum interfaces, propagating inside the gap. The solid line corresponds to the potential profile distributed in the gap for a given value of injected current density J.

are based on a simple 1D electrostatic model, where the 2D pinching effects of self-magnetic field[34] has been completely ignored. While the model is simple, but it provides accurate analytical formulas that have been verified by using particle-in-cell (PIC) simulations in the electrostatic mode.

II. CLASSICAL SHORT-PULSE MODEL

In this paper, we use two models (single-sheet model and equivalent drift space model) to approximate the short-pulse electron flows in a drift space as shown in Fig. 1. In the single-sheet model illustrated in Fig. 1(a), the electron pulse is treated as an infinitesimal (negative) charge sheet, which is injected into a 1D drift space of a gap separation D enclosed by two electrodes held at a potential V_g. The injection velocity is $v_0=\sqrt{2eV_g/m}$. At a position of x within the gap, the electric fields at the front and back of the charge sheet, are $(V_g-V_x)/(D-x)$ and $(V_g-V_x)/x$, respectively. The charge sheet will experience deceleration in the region of $0<x<D/2$, and acceleration in $D/2<x<D$. At the midpoint of the gap $x=D/2$, the velocity of the electron sheet is at its minimum value of v_m, and the electric fields on both sides of the electron sheet will be equal in magnitude, but in opposite directions.

It is clear that the SCL condition occurs when the injected current reaches a critical value for which the minimum velocity v_m at $x=D/2$ equals zero (or the potential becomes zero) resulting a formation of virtual cathode. At this condition, the change in the electric field strength on each side is $\Delta E=\sigma/2\epsilon_0$, where σ is the surface charge density of the electron sheet. At the SCL condition, we have $\Delta E=V_g/(D/2)$, and thus $\sigma=4\epsilon_0 V_g/D$. Based on the single-sheet model, the SCL current density is expressed as $J_M=\sigma/\tau_p$,

$$J_M=\frac{4\epsilon_0 V_g}{D\tau_p}, \tag{3}$$

where τ_p is the pulse length of the injected electron pulse. In normalized form, Eq. (3) becomes

$$\mu=\frac{J_M}{J_d}=\frac{3}{4X_T}, \tag{4}$$

where $\mu=J_M/J_d$ is the normalized short-pulse SCL current density in drift space, and $X_T=\tau_p/T_d$ is the normalized pulse duration. The normalized constants are as follows: J_d is the steady-state SCL current density in drift space, as shown in Eq. (2), and $T_d=3D\sqrt{m/8eV_g}$ is the gap transit time of J_d (the time required for electrons to across the gap at the SCL condition).

Note the simple single-sheet model is not able to recover the steady-state condition: $\mu=3/4$ at $X_T=1$ [see Eq. (4)]. This limitation can, however, be improved by the equivalent drift space model as shown in Fig. 1(b). In this model, we follow the same assumptions as those stated in the single-sheet model, except that finite pulse length ζ of electron pulse is included in the model. In the process of the electron pulse propagation from $x=0$ to $x=D$, the potential at the front and end of the pulse, ϕ_a (at $x=x_a$) and ϕ_c (at $x=x_c$), respectively, would gradually increase and decrease. By solving the Poisson equation and the law of energy conservation, we obtain the electric field E_x as

$$E_x=\frac{d\phi(x)}{dx}=\pm\left[4\frac{J}{\epsilon_0}\sqrt{\frac{m}{2e}}\left(\sqrt{\phi(x)}-\sqrt{\phi_m}\right)\right]^{1/2}, \tag{5}$$

where ϕ_m is the minimum potential. When the electron pulse is at the center of the gap as shown in Fig. 1(b), the potentials at the pulse-vacuum interfaces become $\phi_a=\phi_c=\phi_\zeta$, and the corresponding electric fields (at the interfaces) are

$$E_c=\frac{\phi_\zeta-V_g}{x_c}, \tag{6a}$$

$$E_a=-E_c=\frac{V_g-\phi_\zeta}{D-x_c-\zeta}. \tag{6b}$$

To obtain the short-pulse SCL current density in a drift space, we assume that the space-charge-filled region of $x_c \leqslant x \leqslant x_a$ is an equivalent drift space with a gap spacing of ζ and a potential difference of ϕ_ζ. Thus, the SCL current density for such an equivalent drift space is [according to Eq. (2)]

$$J_M(\zeta,\phi_\zeta)=\frac{32}{9}\epsilon_0\sqrt{\frac{2e}{m}}\frac{\phi_\zeta^{3/2}}{\zeta^2}. \tag{7}$$

By using the Gauss law to relate the surface charge density to the electric fields at the interface, J_M can also be expressed as

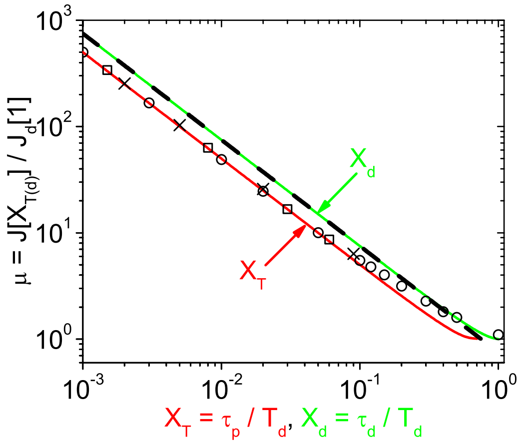

FIG. 2. (Color online) The normalized short-pulse space-charge-limited (SCL) current density $\mu = J_M/J_d$ in a drift space, as a function of the normalized injection pulse duration X_T. The analytical results for the equivalent drift space model with [Eq. (11)] and without [Eq. (9a)] pulse expansion are plotted in solid lines, respectively (bottom and top). The single-sheet model [Eq. (4)] is plotted in dashed line. The symbols corresponds to the PIC simulation results (circles for $V_g=1$ kV and $D=1$ cm; crosses for $V_g=100$ kV and $D=1$ cm; squares for $V_g=1$ kV and $D=10$ cm).

$$J_M = \frac{\epsilon_0}{\tau_d}(E_a - E_c) = \frac{2\epsilon_0}{\tau_d}E_a, \tag{8}$$

where τ_d is the gap transit time of the SCL electrons over a distance of $\zeta = x_a - x_c$. Using Eqs. (5)–(8), we obtain

$$\mu(X_d) = \frac{2}{X_d^3}\left(1 - \sqrt{1 - \frac{3}{4}X_d^2}\right), \tag{9a}$$

$$\bar{\phi}_\zeta^{1/2}(X_d) = \frac{2}{X_d}\left(1 - \sqrt{1 - \frac{3}{4}X_d^2}\right), \tag{9b}$$

$$\bar{\zeta}(X_d) = 2\left(1 - \sqrt{1 - \frac{3}{4}X_d^2}\right), \tag{9c}$$

where $\mu = J_M/J_d$ (also, $\mu = \bar{\phi}_\zeta^{3/2}/\bar{\zeta}^2$) is the normalized short-pulse SCL current density, $\bar{\phi}_\zeta = \phi_\zeta / V_g$ is the normalized potential difference, $\bar{\zeta} = \zeta / D$ is the normalized propagation length, and $X_d = \tau_d / T_d$ is the normalized transit time across the equivalent drift space. At $X_d = 1$, $\bar{\phi}_\zeta = \bar{\zeta} = \mu = 1$, the short-pulse SCL current density recovers the steady-state value.

Note the short-pulse enhancement factor μ from both Eqs. (4) and (9a) are same as the ones predicted from the short-pulse CL law,[26] even though formation of the virtual cathode is at a different position. For drift space, it is near to the center of the gap as compared to near cathode (for a diode). By comparing with PIC simulation (see below), we, however, found that both Eqs. (4) and (9a) overestimate the SCL current value at short-pulse length $X_d < 0.1$ (see Fig. 2). This discrepancy is because the length of the electron pulse in a drift space will expand when it arrives at the midpoint of the gap, where the virtual cathode is formed. This finding indicates that the expanded pulse length τ_d will not equal the initial injection pulse length τ_p.

To include the effect of pulse expansion, we compare the relative strength of the space-charge field near cathode and at the center of the gap. In a drift space, the electrons are injected with high kinetic energy; thus, the space-charge effect near to the cathode is negligible as compared to the center of the gap, where electrons start to be reflected at SCL current condition. To the simplest approximation, we assume that the space-charge effect near to the cathode may be completely ignored, especially for a very short injection pulse, and the electrons will have a transit time of $T_p = D\sqrt{m/2eV_g}$ (without the space-charge effect). On the other hand, the transit time will become $T_d = \frac{3}{2}D\sqrt{m/2eV_g}$ if the space-charge effect is important. Thus, we can relate the pulse injection duration τ_p to the pulse duration at the equivalent drift space mode τ_d by comparing the typical transit time in the drift space between the two cases with and without the space-charge effect, which are defined as T_d and T_p, respectively. In doing so, we establish an approximate relation of $\tau_d = \frac{3}{2}\tau_p$, which is in the normalized form of

$$X_d = \frac{\tau_d}{T_d} = \frac{3}{2}\frac{\tau_p}{T_d} = \frac{3}{2}X_T. \tag{10}$$

Using Eqs. (9a) and (10), the enhancement factor of the short-pulse SCL current density (with pulse expansion) becomes

$$\mu(X_T) = \frac{16}{27X_T^3}\left(1 - \sqrt{1 - \frac{27}{16}X_T^2}\right), \tag{11}$$

which is a function of the normalized injection duration $X_T = \tau_p / T_d$, and it is valid for $X_T < 4/(3\sqrt{3}) \simeq 0.77$.

To verify our analytical results, i.e., Eqs. (4), (9a), and (11), we use a 2D particle-in-cell (PIC) simulation code: OOPIC Pro.[35] The simulations were performed by using the same over-injection method[27] in a planar drift space with a gap separation much smaller than the electrode size (1D model). A finite pulse τ_p of current density J is injected into the gap, and the value of J is increased until the formation of a virtual cathode, which will cause the reflection of electrons. It must be emphasized that all the simulations were conducted in the electrostatic mode to be consistent with the model.

From Fig. 2, we observe that the equivalent drift space models without pulse expansion, i.e., Eq. (9a), is only valid in a small range of relatively large injection pulse: $0.4 \leqslant X_d \leqslant 1$ (see top solid line). In this case, the pulse expansion during its propagation is relatively small, so the pulse injection duration τ_p (or X_T) is approximately equal to τ_d (or X_d). Note the single-sheet model [Eq. (4)] is nearly identical to the equivalent drift space models without pulse expansion for $X_d < 0.5$ (see dashed line). For a rather short injection pulse, i.e., $X_T \leqslant 0.1$, the PIC simulation (bottom solid line) agrees better with the equivalent drift space model with pulse expansion $\mu(X_T)$, as described by Eq. (11). Note the verification has been done over a range of $V_g = 1$ to 100 kV, and $D = 1$ to 10 cm. It is observed that the enhancement (in terms of the steady state SCL current density in drift space) is always proportional to X_T^{-1} for small pulse length, and the emitting charges (per area) is a constant.

III. RELATIVISTIC SHORT-PULSE MODEL

In this section, we attempt to include the relativistic effects when the injected beam energy is higher than the electron rest mass like $U=eV_g/mc^2 \geqslant 1$. Using the same methodology as in the classical short-pulse SCL model, we first derive the short-pulse relativistic SCL law in the drift space,

$$J_{RM} \equiv \frac{2\epsilon_0 mc^3}{e}\frac{G^2(\gamma_\zeta)}{\zeta^2} = \frac{2\epsilon_0}{\tau_{rd}} a_\zeta \frac{\phi_\zeta}{\zeta}, \tag{12}$$

$$a_\zeta = \frac{2G(\gamma_\zeta)}{\gamma_\zeta - 1}\left(\sqrt{\gamma_\zeta^2-1} - \sqrt{\gamma_{m\zeta}^2-1}\right)^{1/2}, \tag{13}$$

where a_ζ and $\gamma_\zeta = 1+e\phi_\zeta/mc^2$ are, respectively, the normalized electric field and the relativistic factor at the beam-vacuum interfaces with a potential of ϕ_ζ, and ζ is the pulse width. Here, $\gamma_{m\zeta}=1+e\phi_{m\zeta}/mc^2$ is the relativistic factor at the location of potential minimum of $\phi_{m\zeta}$ inside the charge-filled region, and $G(u)$ is the maximum value of an integral $[g(u)=\int_{u_m}^{u}(\sqrt{r^2-1}-\sqrt{u_m^2-1})^{-1/2}dr]$, which is obtained by numerically solving $g(u)$ from $u_m=1$ to $u_m=u$.

By setting $\phi_\zeta = V_g$ and $\zeta = D$ in Eq. (12), the gap transit time in the drift space gap for the relativistic SCL electron flows is $T_{rd}=2(D/c)(\sqrt{\gamma_0^2-1}-\sqrt{\gamma_m^2-1})^{1/2}/G(\gamma_0)$, where $\gamma_0 = 1+eU$, $\gamma_m = 1+eU_m$, $U=eV_g/mc^2$, and $U_m=e\phi_m/mc^2 = eV_g/4mc^2$. In terms of the normalized parameters, Eq. (12) becomes

$$\frac{\bar{\zeta}}{X_{rd}} = \frac{G(\gamma_\zeta)}{G(\gamma_0)}\left(\frac{\sqrt{\gamma_0^2-1}-\sqrt{\gamma_m^2-1}}{\sqrt{\gamma_\zeta^2-1}-\sqrt{\gamma_{m\zeta}^2-1}}\right)^{1/2}, \tag{14}$$

where $X_{rd}=\tau_{rd}/T_{rd} \leqslant 1$. Using Eq. (12) and the same equivalent drift space concept, the normalized short-pulse relativistic SCL law in a drift space [in terms of Eq. (2)] is

$$\mu_R \equiv \frac{J_{RM}}{J_d} = \frac{9}{16\sqrt{2}}\frac{G^2(\gamma_\zeta)}{(\gamma_0-1)^{3/2}\bar{\zeta}^2}, \tag{15}$$

which will recover the classical limit at $U \ll 1$. By using Eqs. (13) and (14), and the condition of continuous electric field at the beam-vacuum interfaces [Eq. (6)], we can calculate numerically $\mu_R(X_{rd})$ as a function of X_{rd} and U. Analogous to the classical case, we can also establish a relationship between the relativistic pulse equivalent drift space transit time τ_{rd} (with pulse expansion) and the pulse injection duration τ_{rp} (without pulse expansion), which has a ratio of

$$b(\gamma_0) = \frac{\tau_{rd}}{\tau_{rp}} = \frac{2\sqrt{\gamma_0^2-1}}{\gamma_0 G(\gamma_0)}\left(\sqrt{\gamma_0^2-1}-\sqrt{\gamma_m^2-1}\right)^{1/2}. \tag{16}$$

Thus, the normalized SCL current density $\mu_R(X_{rT})$ can be numerically calculated as a function of the normalized pulse injection duration $X_{rT}=\tau_{rp}/T_{rd}$ for any values of U, as shown in Fig. 3(a) for $U=0.01$ to 100 (top to bottom). For a given U, μ_R increases with the decreasing value of X_{rT}, which also scales as X_{rT}^{-1} at small X_{rT}. In Fig. 3(b), the results are also plotted as a function of U for different $X_{rT}=0.001$ to 1 (long pulse). For a given pulse length, the effect of pulse expansion decreases as U increases, and $\mu_R(X_{rT})$ approaches $\mu_R(X_{rd})$ for $U>10$. At this high U, the transit time with pulse duration (τ_{rd}) due to the space-charge effect is nearly equal to the one without the pulse duration (τ_{rp}), which means Eq. (16) is about 1, indicating $\tau_{rd}=\tau_{rp}$. Thus, the effect of pulse expansion due to the space-charge effect is negligible at large $U>10$, and the variation of electron's velocity, which is approaching speed of light, is not sensitive to space-charge effect.

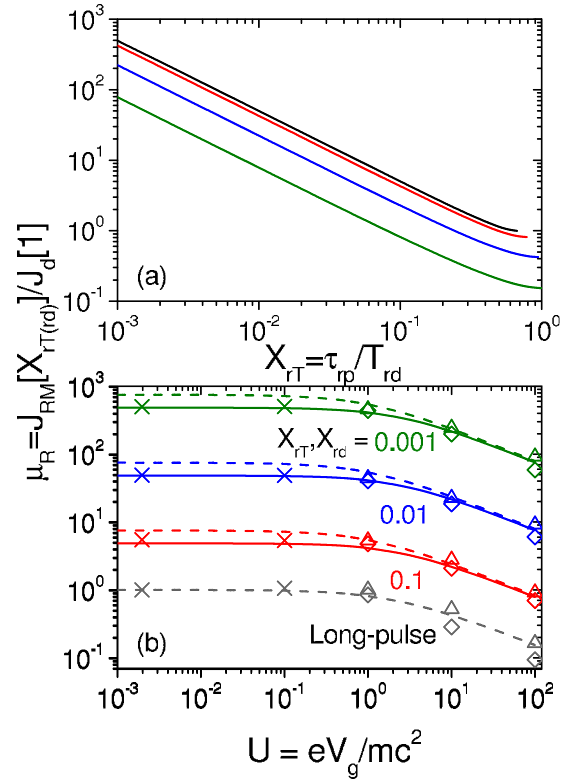

FIG. 3. (Color online) The normalized short-pulse relativistic SCL current density in drift space μ_R, as a function of (a) the normalized pulse injection duration $X_{rT}=\tau_{rp}/T_{rd}$ for $U=0.01$ to 100 (top to bottom), and (b) the normalized electron beam voltage U for $X_{rT}=0.001$ to 1 (long pulse). The solid and dashed lines represent, respectively, the results with and without pulse expansion. The symbols in (b) illustrate the PIC simulation results in the electrostatic mode: (×) is with no adjustment, (△) is with γm_e, and (◇) is with adjustment to have the same transit time to the theoretical value.

From Fig. 3(b), the results indicate that the relativistic effects at $U \geqslant 1$ will suppress the enhancement of the short-pulse effect at a fixed normalized injection pulse duration of X_{rT}. To check this finding, the transit time of a single particle computed by the PIC code (symbols) is compared with the theoretical calculations (lines) in both electrostatic (ES) and electromagnetic (EM) mode at different settings as shown in Fig. 4. It is found that the transit time from the PIC code is not accurate in the ES mode at relativistic energy, which is equal to the transit time at nonrelativistic energy (see cross and circle symbols in Fig. 4). Compared with the calculations, they are only valid up to $U \approx 0.1$. The EM results from PIC code are, however, accurate (see inverted triangle symbols in Fig. 4). This comparison suggests that the particles are not properly *pushed* in the PIC code that we used in the ES mode. In order for us to verify our calculations (based on a simple electrostatic model) by using PIC simulation in the ES mode, we have used the following two methods in chang-

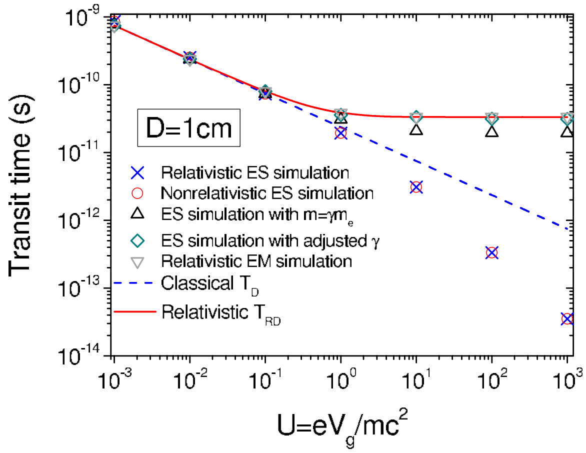

FIG. 4. (Color online) The comparison between the transit time of a single particle in a 1 cm gap obtained from PIC simulation (symbols) and calculations (lines) as a function of $U=0.001$ to 1000.

ing the value of the electron mass in the input file to account for the relativistic effect in the transit time. First, we assume the free electron mass is enhanced by the relativistic factor of $\gamma = 1+U$. In doing so, the transit time of the single particle becomes more accurate (but still somehow smaller) as compared to the calculated results (see triangle symbols in Fig. 4). For further improvement, we adjust the electron mass in order to have the accurate transit time at each value of U (see diamond symbols in Fig. 4). With these modifications, the PIC simulations in the ES mode are able to confirm our calculations of the relativistic short-pulse SCL electron flow in drift space as shown in Fig. 3(b).

IV. SUMMARY

In summary, we have presented a simple 1D short-pulse analytical electrostatic model of the SCL electron flows in a drift space for both classical and relativistic regimes. In the short-pulse limit, the maximum allowed SCL current density that can be transported through the gap is enhanced by a large factor as compared to the long-pulse case, which is verified by using PIC simulation in the electrostatic mode. At a fixed normalized injection pulse length, the enhancement is expected to be suppressed when the electron injection energy is increased to relativistic regime. The effect of pulse expansion due to the space-charge effect is not significant at relativistic energy.

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