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Space charge waves in a two-dimensional electron gas

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ABSTRACT

This paper uses field theory to derive the exact dispersion relation of space charge waves in a two-dimensional electron gas (2DEG) located in a dielectric or a dissimilar dielectric waveguide. It is found that the dispersion of a 2DEG can be modeled accurately using the free-electron sheet model, which is further confirmed by the almost identical polarizability of a 2DEG and of a free-electron sheet with zero drift velocity. Transitions among the well-known 2DEG dispersion, the beam mode in vacuum electronics, and Gould–Trivelpiece mode in plasma physics are demonstrated by varying the 2DEG density and direct current drift velocity. The effects of waveguide dimensions are also presented. Our method is general and can be applied to find the dispersion relation of 2DEG with arbitrary drift velocity (governed by electric field and scattering) in more complex circuits. Our study provides insight into the design of electromagnetic wave devices and circuits involving a 2DEG.

I. INTRODUCTION

A two-dimensional electron gas (2DEG) is formed when electrons are confined to an interface between two different materials, such as the interface in GaN-based heterostructures.[1] Electrons in a 2DEG are free to move in the two in-plane dimensions, but they are tightly confined in the third, transverse direction. As fabrication processes[2] for the GaN-based high-electron-mobility transistors (HEMTs)[3] are mature, the 2DEG density at GaN-based interfaces is highly predictable and uniform. The interaction of a 2DEG in a solid-state heterostructure with a surrounding slow wave circuit holds the potential for high power THz generation and amplification. For high-frequency operation, the traditional vacuum electron beam-based devices would need to be scaled down to microscale dimensions, where inevitable problems, such as strong external magnetic field needed for beam confinement, stringent manufacturing, and alignment tolerances at shorter wavelengths become increasingly challenging. The dense 2DEG formed at material interfaces will provide an ideally confined beam of high-density electrons, where problems pertinent to the vacuum electron beams can be eliminated.

To utilize a 2DEG in electromagnetic wave amplifiers or sources, it is important to understand the behavior of space charge waves in a 2DEG.[4] The dispersion of space charge waves in a 2DEG has been extensively studied in the literature.[4–7] The well-known 2DEG dispersion relation of $\omega^2 \sim \beta$ (ω and β are the wave frequency and wavenumber, respectively) was derived by Stern[4] based on the self-consistent-field treatment of the response of the electron gas, and it has since been considered as the signature of a 2DEG distinguished from waves in traditional three-dimensional bulk materials. On the other hand, thin free-electron sheets are widely used to model electron beams interacting with surrounding circuits in vacuum electronics,[8–15] where the beam mode is described by $\omega = \beta v_0$, with v_0 being the electron drift velocity. The free-electron sheet beams are often assumed to be magnetically confined such that electrons move only along the plane of the sheet. Thus, 2DEG at solid-state material interfaces and free-electron sheet beam in vacuum electronics share similar signatures with electrons being free to move in the in-plane dimensions, but strongly confined in the transverse direction. Despite the very different confinement mechanisms for 2DEG (due to quantum mechanical potential well) and the free-electron beam (due to strong external magnetic field), they are extensively modeled using similar approaches for device applications involving electromagnetic waves,[16–20] including Monte Carlo particle simulations,[21–23] Boltzmann transport theory,[7,24] the hydrodynamic approach,[7,25–27] and particle-in-cell (PIC) simulations.[28] However, the connection of the dispersion relations between the square-root dependence $\omega^2 \sim \beta$ of the 2DEG and the linear dependence of $\omega \sim \beta$ of the free-electron sheet is not well understood, and it is ambiguous if a 2DEG can be modeled as a free-electron sheet beam to study space charge waves and their interaction with the surrounding circuits. Furthermore, it is worthwhile to mention that Stern's theory[4] for 2DEG is based on the assumption of zero drift velocity of the electrons. In this context, there are still important open questions regarding the 2DEG dispersion: Is Stern's theory[4] still valid for 2DEG with finite drift velocity? What is the dependence of the 2DEG dispersion on electron drift velocity? For a 2DEG with a small drift velocity compared to the space charge wave's phase velocity, i.e., a fast space charge wave on the 2DEG with respect to the electron drift velocity, is it possible to achieve wave amplification? This paper attempts to address the first two questions and provide directions that could answer the third.

In this work, we provide the exact dispersion relation for the space charge waves (or plasma waves) in a 2DEG located inside a dielectric or a dielectric-loaded waveguide. The results are compared to those of an ideal free-electron sheet beam with an infinite axial magnetic field. It is explicitly shown that the dispersion relation for a 2DEG can be very accurately modeled by that for a free-electron sheet. The dependence of the dispersion relation on the charge density, direct current (DC) drift velocity, and waveguide dimensions is examined in detail. Transitions among the well-known 2DEG dispersion $\omega^2 \sim \beta$, beam mode in vacuum electronics $\omega \sim \beta$, and Gould–Trivelpiece mode in plasma physics[29] are demonstrated. The results provide useful insights into utilization of 2DEGs in electromagnetic devices and circuits.

II. 2DEG INSIDE A UNIFORM DIELECTRIC

A. Dispersion relation from the polarizability of 2DEG

Consider a longitudinal *total* electric field $\vec{E} = \hat{y}E_0 e^{i\omega t - i\beta y}$ acting on a 2DEG located at $x = 0$, as shown in Fig. 1; the induced polarization is

$$\vec{P} = \epsilon_0 \chi_e \vec{E}\delta(x), \tag{1}$$

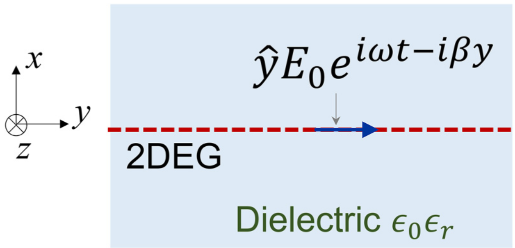

FIG. 1. A 2DEG confined to the *y*–*z* plane inside a uniform dielectric with the direction of space charge wave propagation arbitrarily oriented along the *y*-direction.

where ϵ_0 is the free space permittivity and χ_e is the polarizability. By assuming $T = 0$K, χ_e is obtained by Stern from the self-consistent-field treatment of the response of 2DEG to give[4]

$$\chi_e = \chi_1 + i\chi_2, \tag{2a}$$

$$\chi_1 = G\left[2z - C_- \sqrt{(z-u)^2 - 1} - C_+ \sqrt{(z+u)^2 - 1}\right], \tag{2b}$$

$$\chi_2 = G\left[D_- \sqrt{1-(z-u)^2} - D_+ \sqrt{1-(z+u)^2}\right], \tag{2c}$$

$$C_\pm = \mathrm{sgn}(z \pm u),\ \ D_\pm = 0 \ \text{ if } |z \pm u| > 1, \tag{2d}$$

$$C_\pm = 0,\ \ D_\pm = 1 \ \text{ if } |z \pm u| < 1. \tag{2e}$$

Here, $G = 2m_e^* e^2 N_s/(\epsilon_0 \hbar^2 k_F \beta^3)$, $z = \beta/(2k_F)$, $u = m_e^* \omega/(\beta\hbar k_F)$, where e (<0) is the electron charge, m_e^* is the effective electron mass in the 2DEG, N_s is the number of electrons per unit area, $k_F = m_e^* v_F/\hbar$ is the Fermi wave vector, and v_F is the Fermi velocity. The induced charge perturbation density on the 2DEG is

$$\sigma = -\nabla \cdot \vec{P} = i\beta\epsilon_0\chi_e E_0 \delta(x), \tag{3}$$

where polarization in Eq. (1) due to the total electric field $\vec{E}$ is used.

The presence of the perturbed charge density on the 2DEG would induce electromagnetic fields in the surroundings of the 2DEG. In the case of a 2DEG inside a uniform dielectric as shown in Fig. 1, the induced fields become (see Appendix A for derivation)

$$E_y = Ae^{-\alpha x} e^{i\omega t - i\beta y}, \tag{4a}$$

$$E_x = -\frac{j\beta}{\alpha} Ae^{-\alpha x} e^{i\omega t - i\beta y}, \tag{4b}$$

$$B_z = \frac{j\omega\epsilon_r}{\alpha c^2} Ae^{-\alpha x} e^{i\omega t - i\beta y}, \tag{4c}$$

for $x > 0$, and

$$E_y = Be^{\alpha x} e^{i\omega t - i\beta y}, \tag{5a}$$

$$E_x = \frac{j\beta}{\alpha} Be^{\alpha x} e^{i\omega t - i\beta y}, \tag{5b}$$

$$B_z = -\frac{j\omega\epsilon_r}{\alpha c^2} Be^{\alpha x} e^{i\omega t - i\beta y}, \tag{5c}$$

for $x < 0$. In Eqs. (4) and (5), $\alpha = \sqrt{\beta^2 - \epsilon_r \omega^2/c^2}$, ϵ_r is the relative permittivity of the dielectric, and c is the speed of light in vacuum. The boundary condition at $x = 0$ requires continuity of E_y. Thus, Eqs. (4a) and (5a) give

$$A = B. \tag{6}$$

The normal (x) component of the induced electric field has a jump discontinuity $\sigma/(\epsilon_0\epsilon_r)$ at $x = 0$ according to Gauss's law. Thus, Eqs. (4b) and (5b) give

$$-\frac{i\beta}{\alpha}A - \frac{i\beta}{\alpha}B = \frac{\sigma}{\epsilon_0\epsilon_r}. \tag{7}$$

Remembering that E_0 is the longitudinal electric field experienced by the 2DEG, we have from Eq. (4a),

$$E_0 = A. \tag{8}$$

Equations (3) and (6)–(8) constitute four equations with four unknowns, A, B, σ, and E_0. For a nontrivial solution to exist, the following dispersion relation must be satisfied:

$$2\epsilon_r + \chi_e\sqrt{\beta^2 - \epsilon_r\omega^2/c^2} = 0, \ \textbf{Exact}, \ \textbf{2DEG}, \tag{9}$$

where the polarizability χ_e is given in Eq. (2). Note that Eq. (9) is identical to Eq. (6) of Stern[4] by setting the dielectric constant to be zero. This verifies that applying the widely used field theory in vacuum beam physics (also see Sec. II B)[10,14] gives the same dispersion relation as treating 2DEG as an effective dielectric media. It opens up the possibility of providing a unified field theory to include both the 2DEG dispersion and the vacuum electronics beam mode. As shown later, this unified theory extends the dispersion relation of 2DEG to nonzero drift velocities, where the well-known $\omega^2 \sim \beta$ scaling is derived only for the case of zero drift velocity for 2DEG.

In the long wavelength limit, i.e., $u \gg 1$, Eq. (2) is simplified to read as $\chi_e \simeq -z/u^2$, the dispersion relation in Eq. (9) becomes

$$\beta^2 = \epsilon_r\frac{\omega^2}{c^2} + \left(\frac{2\epsilon_r\omega^2}{\omega_a^2}\right)^2, \tag{10}$$

which is identical to Eq. (7) of Ref. 4. In Eq. (10), $\omega_a^2 \equiv (e^2N_s)/(m_e^*\epsilon_0)$. As shown later, Eq. (10) is the same as the free-electron sheet dispersion, as in Eq. (17). In the "not so long" wavelength limit, χ_e in Eq. (2) is Taylor expanded to the next order to yield, $\chi_e \simeq -\frac{z}{u^2}\left(1 + \frac{3}{4}\frac{1}{u^2}\right)$, and Eq. (9) becomes

$$\omega^2 = \frac{\omega_a^2}{2\epsilon_r}\beta + \frac{3}{4}\beta^2 v_F^2, \tag{11}$$

which is identical to Eq. (8) of Ref. 4 and Eq. (2.44) of Ref. 5 and is the well-known 2DEG dispersion in the literature, showing a square-root dependence of wave frequency on wave vector. Note that, the same method can be used to derive the "hot-tube" dispersion of 2DEG for different circuit surroundings, including in a waveguide with dissimilar dielectrics or with a slow wave structure.

An example of the dispersion relation for a 2DEG inside a dielectric with uniform permittivity is shown in Fig. 2. It is clear that the two limiting relations, Eqs. (10) and (11), are excellent approximations of the exact dispersion in Eq. (9) that shows a scaling of $\omega \sim \sqrt{\beta}$.

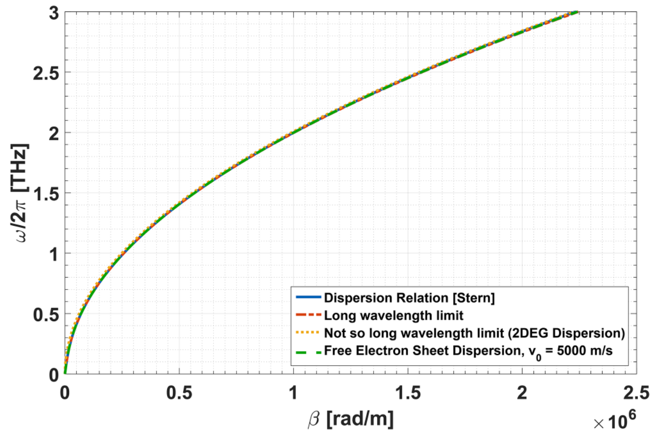

FIG. 2. Dispersion relation for a 2DEG in a uniform dielectric. In the calculation, we use electron charge density $N_s = 2 \times 10^{17}$ 1/m^2, Fermi energy $E_F = 0.24$ eV, temperature $T = 0$ K, relative dielectric permittivity $\epsilon_r = 10$, and effective electron mass $m_e^*/m_e = 0.2$, with m_e being the electron rest mass. The results are calculated from the dispersion relation [Eq. (9)], the long wavelength limit [Eq. (10)], the "not so long" wavelength limit [Eq. (11)], and the free-electron sheet dispersion [Eq. (16)].

B. 2DEG represented by a free-electron sheet

We next consider the representation of the 2DEG by an infinitesimally thin free-electron sheet, which is widely used to study beam-circuit interaction in vacuum tubes.[10,11,14,15] As shown in Fig. 3, a longitudinal *total* electric field $\vec{E} = \hat{y}E_0e^{i\omega t - i\beta y}$ is acting on the free-electron sheet, which is with a constant drift velocity v_0. The charge perturbation density and current perturbation density on the beam are $\rho(x, y, t) = \sigma e^{i\omega t - i\beta y}\delta(x)$ and $\vec{J}(x, y, t) = \hat{y}Ke^{i\omega t - i\beta y}\delta(x)$, respectively. Linearizing the continuity equation $\nabla \cdot \vec{J} + \partial\rho/\partial t = 0$ relates the surface-charge density σ and the

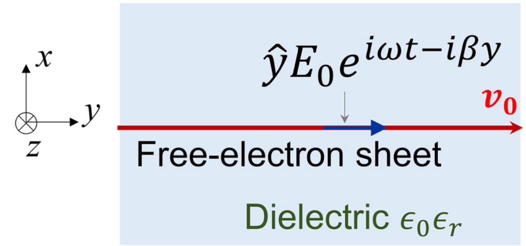

FIG. 3. A 2DEG represented by a free-electron sheet inside a uniform dielectric. The free-electron sheet has a drift velocity of v_0.

surface current K by

$$K = \omega\sigma/\beta. \tag{12}$$

The force law, $(\partial/\partial t + v\partial/\partial y)\vec{v} = e\vec{E}/m_e^*$, is linearized to obtain

$$i(\omega - \beta v_0)v = eE_0/m_e^*, \tag{13}$$

and the linearized current continuity equation gives

$$K = \sigma v_0 + \sigma_0 v, \tag{14}$$

where $\sigma_0 = eN_s$ is the unperturbed surface-charge density of the free-electron sheet and v_0 is the unperturbed drift velocity. From Eqs. (12)–(14), we can obtain

$$\sigma = \epsilon_0 E_0 \frac{\omega_a^2\beta}{i(\omega - \beta v_0)^2}. \tag{15}$$

The presence of the perturbed charge density on the free-electron sheet would induce electromagnetic fields in the surroundings of the sheet, which can also be described by Eqs. (4) and (5). Following the same procedure outlined in Sec. II A, the surface-charge density in Eq. (15) is solved together with the boundary conditions in Eqs. (6)–(8) to give the following dispersion relation

$$(\omega - \beta v_0)^2 = \frac{\omega_a^2}{2\epsilon_r}\sqrt{\beta^2 - \epsilon_r\frac{\omega^2}{c^2}}, \ \textbf{Exact}, \ \textbf{free-electron sheet}. \tag{16}$$

In the special case of $v_0 = 0$, Eq. (16) becomes

$$\omega^2 = \frac{\omega_a^2}{2\epsilon_r}\sqrt{\beta^2 - \epsilon_r\frac{\omega^2}{c^2}}, \tag{17}$$

which is the same as Eq. (10), the 2DEG dispersion in the long wavelength limit, derived using the polarizability of Stern.[4] Equation (16) is also plotted in Fig. 2, for the case of $v_0 = 5000$ m/s, which almost overlaps with the results from a 2DEG. This suggests that the space charge waves on a 2DEG can be modeled very accurately by the free-electron sheet model, for finite drift velocity. In the case of a small drift velocity of $v_0 = 5000$ m/s ($\ll$ the phase velocity of the space charge waves on the 2DEG or free-electron sheet), v_0 has little effect in the dispersion relation of the given free-electron sheet compared to the case of zero drift velocity. Note from Fig. 2 that in the parameter regime presented, the phase velocity of the space charge waves on the 2DEG or free electron sheet is on the order of 10^7 m/s, much larger than the drift velocity of 5000 m/s. As drift velocity increases and approaches the phase velocity of the space charge waves (e.g., in Fig. 5), the well-known 2DEG dispersion scaling $\omega \sim \sqrt{\beta}$ will not hold anymore, and Eq. (16) needs to be used to account for the effects of nonzero drift velocity.

The excellent agreement between the dispersion relations from 2DEG in Eq. (9) and the free electron sheet in Eq. (16) indicates that the perturbed charge density in the two systems matches closely to each other. By comparing Eqs. (3) and (15), we can obtain an expression for the polarizability of a free-electron sheet with a drift velocity v_0,

$$\chi_{\text{free-electron-sheet}} = -\frac{\omega_a^2}{(\omega - \beta v_0)^2}. \tag{18}$$

Figure 4 shows the comparison of the polarizability for the 2DEG in Eq. (2) and the free-electron sheet in Eq. (18). The excellent agreement at small v_0 further confirms that 2DEG may be modeled accurately by a free-electron sheet for the space charge wave dispersion relation. The polarizability of the drifting free-electron sheet can also be derived directly from the force law, as seen in Appendix B.

The dispersion relation for space charge waves on a free-electron sheet inside a uniform dielectric is shown in Fig. 5 for different drift velocities v_0, calculated from Eq. (16). As DC drift velocity increases from $v_0 = 0$ to $v_0 = 5 \times 10^7$ m/s, the dispersion scaling changes from $\omega^2 \sim \beta$ to $\omega \sim \beta v_0$, where the square-root dependence is the well-known characteristic of the "2DEG dispersion," and the linear dependence is the "beam mode" in vacuum electronics.[9,10,12] Note that the $v_0 \to 0$ case is also known as the Gould–Trivelpiece mode in plasma physics,[19,29] which states that, when a plasma is of *finite transverse* cross section, space charge waves may propagate even in the absence of a drift motion or thermal velocities of the plasma. Note that while a wide range of drift velocity up to $v_0 = 5 \times 10^7$ m/s is used in Fig. 5 for theoretical completeness, in practice, the maximum achievable drift velocity in a 2DEG is limited by the maximum sustainable electric field in the device, scattering, and velocity saturation.

The effect of charge density on the dispersion relation in Eq. (16) is shown in Fig. 6. As 2DEG density N_s increases, the

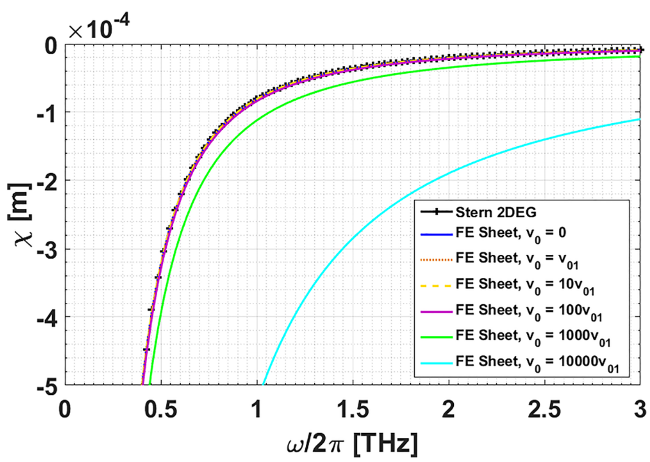

FIG. 4. Comparison of the polarizability of a 2DEG [Eq. (2)] and of a free-electron (FE) sheet for a different drift velocity v_0 [Eq. (18)]. In the calculation, we use electron charge density $N_s = 2 \times 10^{17}$ 1/m^2, $v_{01} = 5000$ m/s, and effective electron mass $m_e^*/m_e = 0.2$, with m_e being the electron rest mass.

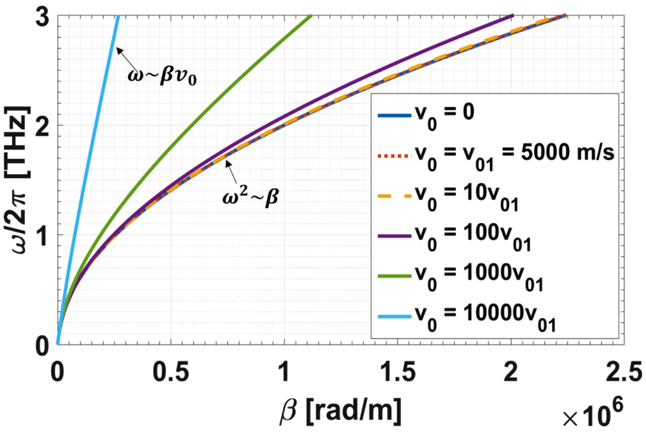

FIG. 5. Dispersion relation for a free-electron sheet in a uniform dielectric for a different drift velocity v_0, calculated from Eq. (16). In the calculation, we use electron charge density $N_s = 2 \times 10^{17}$ 1/m^2, relative dielectric permittivity $\epsilon_r = 10$, and effective electron mass $m_e^*/m_e = 0.2$, with m_e being the electron rest mass.

dispersion curve shifts from the square-root dependence $\omega^2 \sim \beta$ (i.e., "2DEG dispersion") to $\omega \sim \beta c/\sqrt{\epsilon_r}$, which is essentially the light line in a dielectric. This is expected because Eq. (16) can be simplified to obtain $\omega \approx \beta c/\sqrt{\epsilon_r}$ in the limit of $\omega_a^2 \equiv (e^2 N_s)/(m_e^* \epsilon_0) \to \infty$.

III. 2DEG INSIDE A DIELECTRIC WAVEGUIDE

We next consider a 2DEG inside a waveguide with dissimilar dielectrics, as shown in Fig. 7. The perturbed charge density would induce electromagnetic fields inside the waveguide. Following the same procedure as outlined in Appendix A, the induced fields for $0 < x < a$ in Fig. 7 read[10]

$$E_y = A\sin(p_1 x)e^{i\omega t - i\beta y}, \tag{19a}$$

$$E_x = -A\frac{j\beta}{p_1}\cos(p_1 x)e^{i\omega t - i\beta y}, \tag{19b}$$

$$B_z = A\frac{j\omega\epsilon_{r1}}{c^2 p_1}\cos(p_1 x)e^{i\omega t - i\beta y}, \tag{19c}$$

with $p_1^2 = \epsilon_{r1}\omega^2/c^2 - \beta^2$; and for $a < x < b$, the fields are

$$E_y = B\sin[p_2(x-b)]e^{i\omega t - i\beta y}, \tag{20a}$$

$$E_x = -B\frac{j\beta}{p_2}\cos[p_2(x-b)]e^{i\omega t - i\beta y}, \tag{20b}$$

$$B_z = B\frac{j\omega\epsilon_{r2}}{c^2 p_2}\cos[p_2(x-b)]e^{i\omega t - i\beta y}, \tag{20c}$$

with $p_2^2 = \epsilon_{r2}\omega^2/c^2 - \beta^2$. The dimensions a, b, and the relative permittivity ϵ_{r1} and ϵ_{r2} of the dielectrics are defined in Fig. 7. The boundary conditions at the perfectly conducting (PEC) plates, namely, $E_y = 0$ at $x = 0$ and $x = b$, are satisfied by Eqs. (19a) and (20a). The boundary condition at $x = a$ requires continuity of E_y. Thus, Eqs. (19a) and (20a) at $x = a$ give

$$A\sin(p_1 a) = B\sin[p_2(a-b)]. \tag{21}$$

The normal (x) component of the displacement field $\vec{D} = \epsilon\vec{E}$ has a jump discontinuity because of the perturbed charge density σ at $x = a$ according to Gauss's law. Thus, Eqs. (19b) and (20b) give

$$-B\frac{j\beta\epsilon_0\epsilon_{r2}}{p_2}\cos[p_2(a-b)] + A\frac{j\beta\epsilon_0\epsilon_{r1}}{p_1}\cos(p_1 a) = \sigma. \tag{22}$$

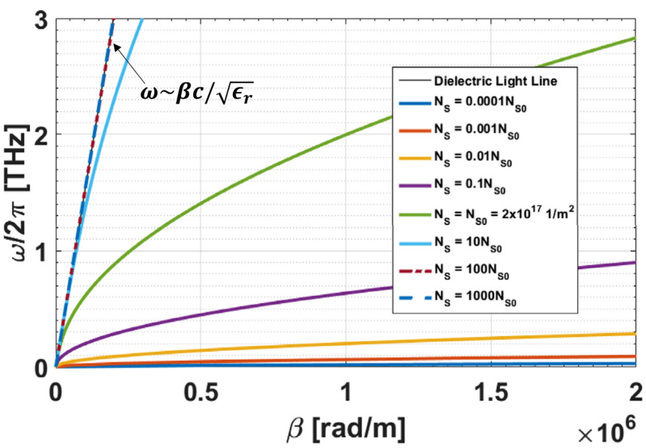

FIG. 6. Dispersion relation for a free-electron sheet in a uniform dielectric for a different charge density, calculated from Eq. (16). In the calculation, we use $v_0 = 5000$ m/s, relative dielectric permittivity $\epsilon_r = 10$, and effective electron mass $m_e^*/m_e = 0.2$, with m_e being the electron rest mass.

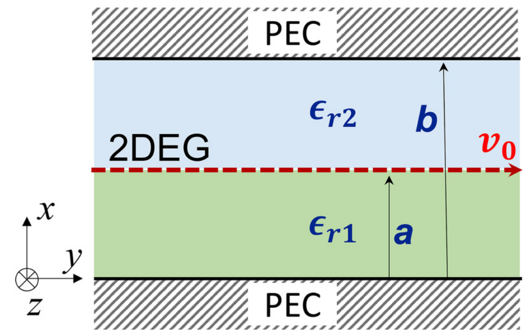

FIG. 7. A 2DEG inside a waveguide with dissimilar dielectrics. PEC stands for a perfect electrical conductor.

The tangential electric field E_0 experienced by the 2DEG can be obtained from Eq. (19a),

$$E_0 = A\sin(p_1 a). \tag{23}$$

The dispersion relation of a 2DEG in a dissimilar dielectric waveguide in Fig. 7 can be found by combining Eqs. (21)–(23) with the perturbed charge density from either Eq. (3) or Eq. (15), which gives

$$\frac{\epsilon_{r1}}{p_1}\cot(p_1 a) - \frac{\epsilon_{r2}}{p_2}\cot[p_2(a-b)] = \chi_e, \textbf{Exact, 2DEG in waveguide}, \tag{24a}$$

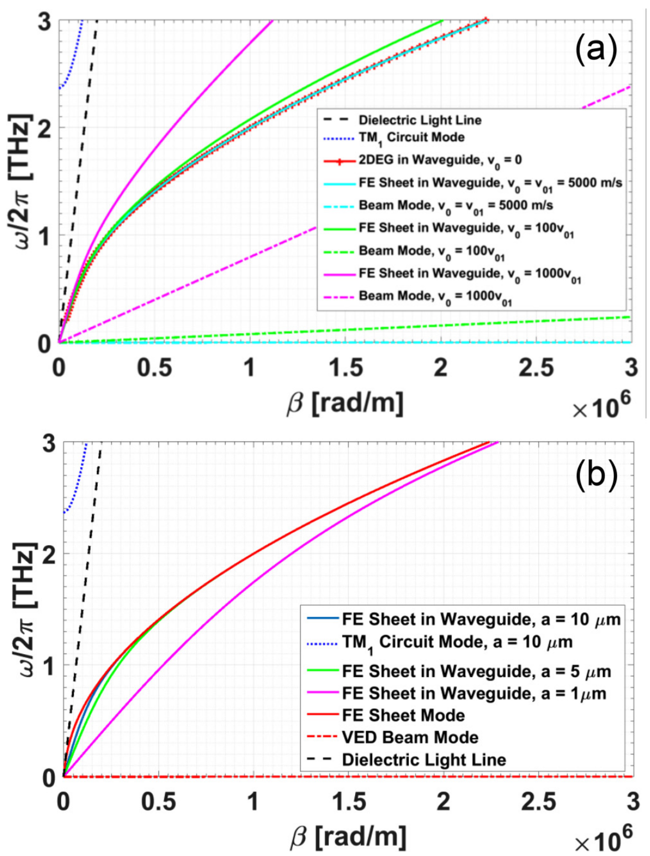

FIG. 8. Dispersion relation for a 2DEG in a dielectric waveguide, calculated from Eq. (24a) for $v_0 = 0$ and from Eq. (24b) for (a) different drift velocity v_0, with $b = 2a = 20\,\mu$m, and (b) different $b = 2a$ with $v_0 = 5000$ m/s. In the calculation, we use electron charge density $N_s = 2 \times 10^{17}$ 1/m^2, relative dielectric permittivity $\epsilon_{r1} = \epsilon_{r2} = 10$, and effective electron mass $m_e^*/m_e = 0.2$, with m_e being the electron rest mass. Also shown are the dielectric light line, $\omega = \beta c/\sqrt{\epsilon_r}$; the vacuum electronics beam mode, $\omega = \beta v_0$; the cold-tube waveguide mode, Eq. (25); and the dispersion relation in an infinite dielectric (FE sheet mode), Eq. (16).

$$\frac{\epsilon_{r1}}{p_1}\cot(p_1 a) - \frac{\epsilon_{r2}}{p_2}\cot[p_2(a-b)] = -\frac{\omega_a^2}{(\omega-\beta v_0)^2},$$
$$\textbf{Exact, free}-\textbf{electron sheet in wave guide}, \tag{24b}$$

where the polarizability χ_e in Eq. (24a) is given in Eq. (2), which is applicable only for $v_0 = 0$. Note that Eq. (24b) recovers Eq. (31) of Ref. 10 if $\epsilon_{r1} = \epsilon_{r2} = 1$. If $\chi_e = 0$ or $\omega_a = 0$, i.e., there is no 2DEG present, Eq. (24) reduces to the cold-tube dispersion of a dissimilar dielectric waveguide,

$$\frac{\epsilon_{r1}}{p_1}\cot(p_1 a) - \frac{\epsilon_{r2}}{p_2}\cot[p_2(a-b)] = 0. \tag{25}$$

Figure 8(a) shows the dispersion relation calculated from Eq. (24a) for $v_0 = 0$ and from Eq. (24b) with a different drift velocity v_0. Figure 8(b) shows the dispersion relation for different waveguide dimensions a. It is clear that, in the chosen parameter regime, the space charge wave in 2DEG is a fast wave with respect to the small constant drift velocity v_0, since the space charge wave dispersions are above their corresponding vacuum beam mode $\omega = \beta v_0$, indicating a faster phase velocity than v_0. In all the cases shown, because of the "fast wave" nature, there is no wave growth, i.e., no imaginary β in the chosen set of parameters. As shown in Fig. 8(b), comparing with 2DEG in an infinite dielectric [Eq. (16)], the effects of waveguide walls become important only when the dimension $b = 2a \le 20\,\mu$m. When $a = 10\,\mu m$ and $5\,\mu m$, the effects of waveguide walls are more significant in the small wavenumber regime when $\beta < 0.5 \times 10^6$ rad/m; and its effect extends to larger β when the waveguide dimension reduces to $a = 1\,\mu$m.

IV. CONCLUSION

This work derives a general dispersion relation for a 2DEG with arbitrary drift velocity (governed by electric field and scattering). It provides a unified theory that includes the dispersion relations for 2DEG, the planar Trivelpiece–Gould mode, and the free-electron beam mode, depending on the drift velocity and density of the electrons. In particular, using the field theory by consistently solving Maxwell's equations and the 2DEG polarizability, we have re-derived the exact dispersion relation for the space charge waves (or plasma waves) for a 2DEG in a uniform dielectric. It is found that the space charge waves on a 2DEG can be modeled accurately using the free-electron sheet model. This is further verified by the excellent agreement on the polarizability of a 2DEG and a free-electron sheet with zero drift velocity. Transitions among the well-known "2DEG dispersion" $\omega^2 \sim \beta$, "beam mode" in vacuum electronics $\omega = \beta v_0$, and Gould–Trivelpiece mode in plasma physics are demonstrated by varying the 2DEG density and DC drift velocity. The verification of the results in various limits provides a theoretical foundation for modeling 2DEG using the free-electron sheet model for arbitrary electron drift velocity.

It is found that when the drift velocity is small (compared to the phase velocity of the space charge wave), the dispersion relation $\omega^2 \sim \beta$ for 2DEG holds and the effect of drift velocity is not important. However, as the drift velocity increases, the well-known 2DEG dispersion scaling $\omega^2 \sim \beta$ becomes invalid and the general dispersion relation [i.e., Eqs. (16) and (24a)] needs to be used to model the 2DEG dispersion.

The exact dispersion relation for a 2DEG in a dissimilar dielectric waveguide is also derived. Our formulation provides a general framework to find the dispersion relation of 2DEG of arbitrary drift velocity with different circuit surroundings. It is shown that space charge waves in a 2DEG are fast waves with respect to the electron's drift velocity. As a result, wave growth may only be possible if 2DEG has a sufficiently large drift velocity such that the "beam mode" $\omega = \beta v_0$ intersects with the circuit mode. The results provide useful insights on the possible development of electromagnetic wave amplifiers using 2DEG. More generally, as the subject touches upon three areas, solid state, plasma, and beam physics, our study could be useful to a broad range of research fields and applications involving space charge waves.

It is worthwhile to note that Eq. (16) is a quadratic equation, implying two solutions in general, and only one is plotted in Figs. 5, 6, and 8. Once the drift velocity gets above the free-electron sheet's plasma velocity, it is possible that there may be a second wave that is slower than v_0. Whether or not this possible slow mode is physically realizable or useful in producing gain is still to be determined.

While our results demonstrate the utilization of the free-electron sheet model to study 2DEG, theoretically, it is still desirable to formulate a 2DEG dispersion relation using Stern's approach[4] with the inclusion of drift velocity. Such a formulation would further verify our approach in this paper. Future studies may also consider the effects of collisions and thermal effects in the 2DEG on the space charge waves. The dispersion relations may be compared with those from Pierce's formulation. Our proposed formulation may also be applied to study more complex circuits, such as a nearby slow wave structure. In addition, it would be interesting to experimentally measure the parametric dependence of the 2DEG dispersion on the electron density and electron drift velocity predicted in the work.

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AUTHOR DECLARATIONS

Conflict of Interest

The authors have no conflicts to disclose.

DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author upon reasonable request.

APPENDIX A: THE FIELD SOLUTION AROUND A 2DEG

The wave equation in an ideal dielectric is read as

$$\left(\nabla^2 - \frac{\epsilon_r}{c^2}\frac{\partial^2}{\partial t^2}\right)\vec{E} = 0, \tag{A1}$$

which has a transverse magnetic (TM) solution in the space above and below the 2DEG in Fig. 1, assumed in the form of

$$\vec{E} = \hat{x}E_x(x, y, t)e^{i\omega t - i\beta y - ipx} + \hat{y}E_y(x, y, t)e^{i\omega t - i\beta y - ipx}. \tag{A2}$$

Putting the above solution Eq. (A2) into Eq. (A1), we find

$$p^2 = \frac{\epsilon_r}{c^2}\omega^2 - \beta^2. \tag{A3}$$

Assuming the electric field decays away from the 2DEG, p becomes imaginary, i.e., $p = \pm i\alpha$, with $\alpha = \sqrt{\beta^2 - \epsilon_r\omega^2/c^2}$. Thus, the field components E_x, E_y, and $B_z \propto e^{\pm\alpha x}e^{i\omega t - i\beta y}$, where the $\pm$ sign is for the region $x < 0$ and $x > 0$, respectively. By assuming E_y in the form of Eqs. (4a) and (5a), the solution for E_x and B_z in Eqs. (4b), (4c), and (5b), 5(c) can be easily obtained from Ampere's law, $\nabla \times \vec{B} = (\epsilon_r/c^2)\partial\vec{E}/\partial t$.

APPENDIX B: POLARIZABILITY OF A FREE-ELECTRON SHEET

The force law for a free-electron sheet with a constant drift velocity v_0 is

$$\frac{D\vec{s}}{Dt} = \vec{v}, \quad \frac{D\vec{v}}{Dt} = \frac{e}{m_e^*}\vec{E}, \tag{B1}$$

where $\vec{s}$ and $\vec{v}$ are the displacement and velocity of the electrons due to the electric field $\vec{E}$, and $D/Dt = \partial/\partial t + v_0\partial/\partial y$ is the total derivative. Assuming $\vec{s}$, $\vec{v}$, and $\vec{E} \propto e^{i\omega t - i\beta y}$, from Eq. (B1), we obtain

$$s = -\frac{e}{(\omega - \beta v_0)^2 m_e^*}E, \tag{B2}$$

where s and E are the magnitudes of $\vec{s}$ and $\vec{E}$, respectively. The displaced electrons contribute to the macroscopic polarization[30] of the free-electron sheet,

$$\vec{P} = -N_S|e|\vec{s} = \epsilon_0\chi_{free-electron\ sheet}\vec{E}. \tag{B3}$$

Putting Eq. (B3) into Eq. (B2), we obtain the polarizability of the free-electron sheet,

$$\chi_{free-electron-sheet} = -\frac{\omega_a^2}{(\omega - \beta v_0)^2}, \tag{B4}$$

where we have used $\omega_a^2 \equiv (e^2N_s)/(m_e^*\epsilon_0)$. Equation (B4) is identical to Eq. (18).

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